Radiation-matter interactions

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Introduction

The description of the interaction of particles with matter is a vast domain...

In this 3h lecture I will not spend times on detailed calculations, etc...
but at the end I hope you'll have good ideas of the things that occurs to particles when they travel through matter...

You'll find lot of informations on the web...

http://pdg.lbl.gov/2012/reviews/rpp2012-rev-passage-particles-matter.pdf

Almost all of these slides have been grabbed from others (thanks to them)

Discovery of the positron e^+

1932 C.D. Anderson :
Particle with positive curvature and minimum ionisation
(size of the droplets)

Track length incompatible with a proton in the air, mass incompatible with a proton

Energy loss in a
6 mm of Pb : **compatible with that of electron**

**Hypothesis (discovery !) :
particle with mass ~m_e
and charge +1, the positron**

First anti-particle

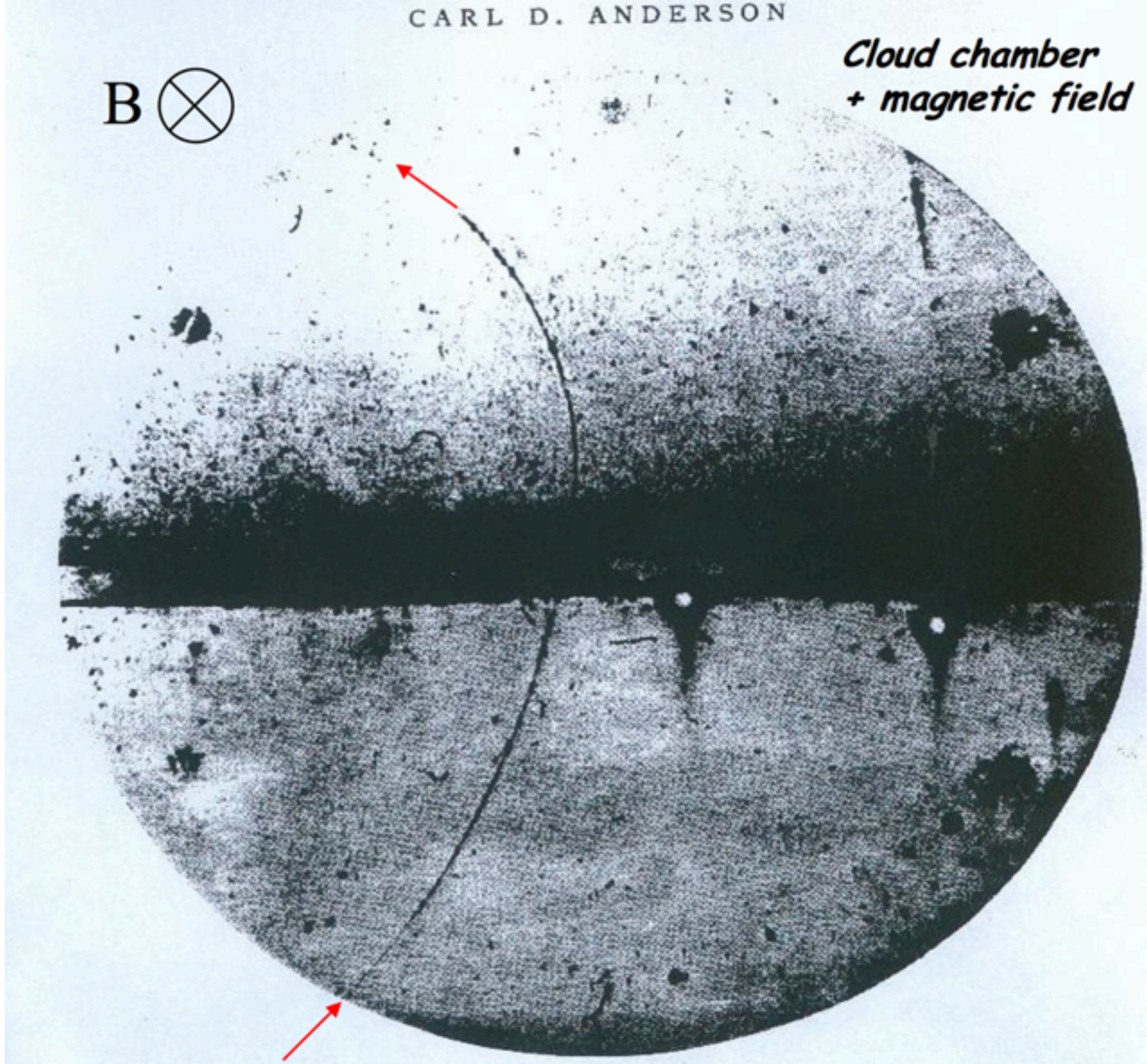


FIG. 1. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plat[e] and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter pat[h] is at least ten times greater than the possible length of a proton path of this curvature.

Units and conventions

$$E^2 = \vec{p}^{\,2}c^2 + m_0^2c^4$$

energy E : measured in **eV**

momentum p : measured in eV/c or **eV**

mass m_0 : measured in eV/c^2 or **eV**

$$\beta = \frac{v}{c} \quad (0 \le \beta < 1) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad (1 \le \gamma < \infty)$$

$$E = m_0\gamma c^2 \qquad p = m_0\gamma\beta c \qquad \beta = \frac{pc}{E}$$

http://www.nature.com/news/2004/040105/images/bee_180.jpg

from C. Joram, SSL 2003

1 eV is a small energy.

1 eV $= 1.6 \cdot 10^{-19}$ J

$m_{\text{bee}} = 1\text{g} = 5.8 \cdot 10^{32}$ eV/c^2

$v_{\text{bee}} = 1$ m/s $\Rightarrow E_{\text{bee}} = 10^{-3}$ J $= 6.25 \cdot 10^{15}$ eV

$E_{\text{LHC}} = 14 \cdot 10^{12}$ eV

However,

LHC has a total stored beam energy

10^{14} protons x $14 \cdot 10^{12}$ eV $\sim 10^8$ J

or, if you like,

one 100 T truck

at 100 km/h

Fixed target vs. collider

Many fixed target experiments up to 60-s (1st by Rutherford),
then collider experiments dominate

$s =$ « m^2 » $= (E_1+E_2)^2 - (p_1+p_2)^2$

Fixed target

$$\longrightarrow \quad \bigcirc$$

$$(\vec{p}, E) \quad (0, m)$$

$$s = (E+m)^2 - p^2$$
$$= 2m(E+m) \sim 2mE$$

Collider

$$\longrightarrow \; \longleftarrow$$

$$(\vec{p}, E) \quad (-\vec{p}, E)$$

Energy, available in the center of mass system : √s

$$s = 4E^2$$

Detector more complicated / large

Q: Why ?

With the same beam energy (but using **two beams**), the center of mass system energy, available to create new particles, is larger with colliders. (interaction on the fixed target used for secondary beams production)

Fixed target vs. collider

In the collision of two particles of masses m_1 and m_2 the total center-of-mass energy can be expressed in the Lorentz-invariant form

$$E_{\rm cm} = \left[(E_1 + E_2)^2 - (\boldsymbol{p}_1 + \boldsymbol{p}_2)^2\right]^{1/2} ,$$
$$= \left[m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2\cos\theta)\right]^{1/2} , \qquad (39.2)$$

where θ is the angle between the particles. In the frame where one particle (of mass m_2) is at rest (lab frame),

$$E_{\rm cm} = (m_1^2 + m_2^2 + 2E_{1\,\rm lab}\,m_2)^{1/2} . \qquad (39.3)$$

The velocity of the center-of-mass in the lab frame is

$$\boldsymbol{\beta}_{\rm cm} = \boldsymbol{p}_{\rm lab}/(E_{1\,\rm lab} + m_2) , \qquad (39.4)$$

where $\boldsymbol{p}_{\rm lab} \equiv \boldsymbol{p}_{1\,\rm lab}$ and

$$\gamma_{\rm cm} = (E_{1\,\rm lab} + m_2)/E_{\rm cm} . \qquad (39.5)$$

The c.m. momenta of particles 1 and 2 are of magnitude

$$p_{\rm cm} = p_{\rm lab}\frac{m_2}{E_{\rm cm}} . \qquad (39.6)$$

For example, if a 0.80 GeV/c kaon beam is incident on a proton target, the center of mass energy is 1.699 GeV and the center of mass momentum of either particle is 0.442 GeV/c.

Which particles do we see in the detector

« Truly » elementary particles :

Leptons spin = 1/2			
SAVEUR		masse GeV/c^2	Charge électriq.
ν_e	neutrino électron.	$<1\times10^{-8}$	0
e	électron	0.000511	−1
ν_μ	neutrino muon	<0.0002	0
μ	muon	0.106	−1
ν_τ	neutrino tau	<0.02	0
τ	tau	1.7771	−1

Quarks spin = 1/2			
SAVEUR		masse GeV/c^2	Charge électriq.
u	up	0.003	2/3
d	down	0.006	−1/3
c	charm	1.3	2/3
s	strange	0.1	−1/3
t	top	175	2/3
b	bottom	4.3	−1/3

Neutrinos ν_i can be seen in the dedicated detector only, or sometimes indirectly. Probability of interaction $P_{Int.}$ with matter is small.

Free quarks have not been observed.
Quarks form hadrons : mesons ($q\bar{q}$) or baryons (qqq).
cf. QCD

« Initial » (diagram) quarks can be probed via measurement of jets.

Examples : $\pi^{+/-}$, p, n, $K^{+/-}$, etc.

Z^0, $W^{+/-}$, H decay before they reach active part of the detector

force électrofaible spin = 1		
Nom	Masse GeV/c^2	charge électriq.
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Invisible : *jets*

interaction forte spin = 1		
Nom	Masse GeV/c^2	charge électriq.
g gluon	0	0

+ Higgs
+ Super-Symmetric partners ?

Summary

Measure stable and quasi-stable particles (e, γ, μ, π, K, p, n, ν) :
Kinematics (momentum and/or energy)
The way particle interacts with / passes through detectors

Main goal of instrumentation :
Precisely/fast **measure kinematics** of (quasi-) stable particles
Unambiguously/fast **identify** them

For that :
We study **how particles interact with the matter**
and
We choose the **detector technologies** that match the physics tasks

(Heavy) charged particles interaction with matter

« Heavy » charged particles :

those with $M \gg m_e$ (~0.5 MeV/c^2) :

m_μ ~ 106 MeV/c^2

$m_{\pi+/-}$ ~ 140 MeV/c^2

$m_{K+/-}$ ~ 494 MeV/c^2

m_p ~ 938 MeV/c^2

All these charged particles visible in our detector !

« Light » particles : e^-, e^+

And also photon, treated separately.

(Heavy) charged particles interaction with matter

Energy (kinetic) loss by Coulomb interaction with the atoms/electrons :

- Excitation : the atom (or molecule) is excited to a higher level

  atom* → atom + γ

  low energy photons of de-excitation

  ➔ light detection

- Ionization : the electron is ejected from the atom

  electron / ion pair

  ➔ charge detection

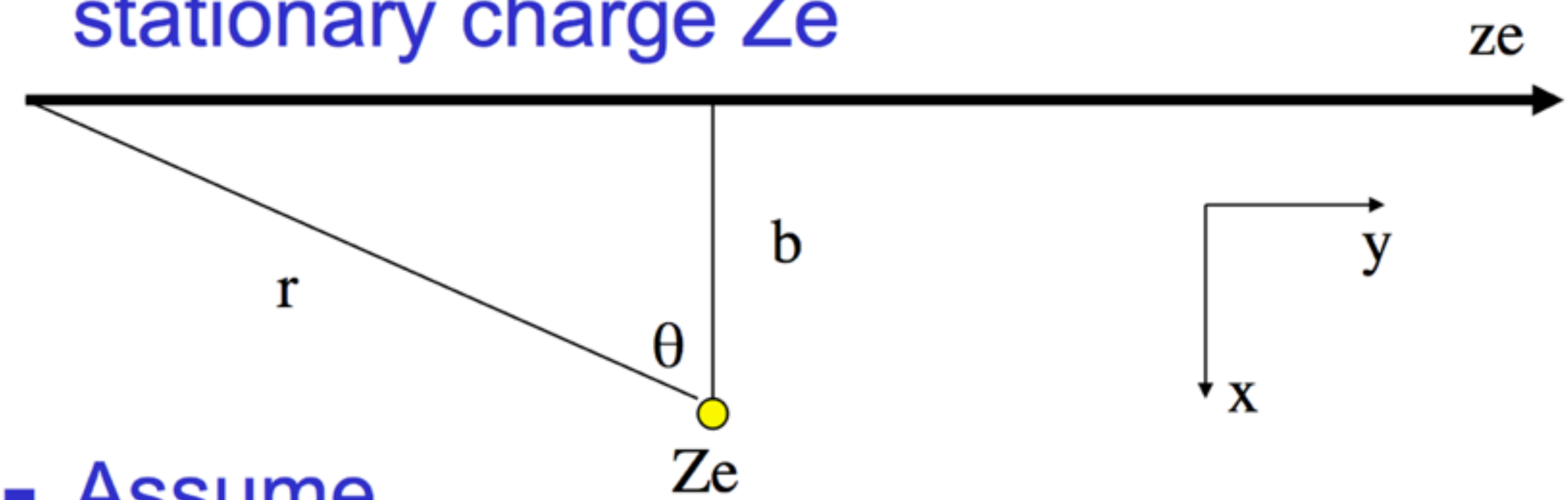
- Instead of ionization/excitation real photon can be produced under certain conditions

  ➔ Cherenkov or Transition radiation

  Contribute very little to the energy loss (< 5%), can be neglected but they are used for particle ID

Bethe-Bloch formula

- Consider particle of charge ze, passing a stationary charge Ze



- Assume
  - Target is non-relativistic
  - Target does not move
- Calculate
  - Energy transferred to target   (separate)

Bethe-Bloch formula

- Force on projectile

$$F_x = \frac{Zze^2}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Zze^2}{4\pi\varepsilon_0 b^2}\cos^3\theta$$

- Change of momentum of target/projectile

$$\Delta p = \int_{-\infty}^{\infty} dt F_x = \frac{Zze^2}{2\pi\varepsilon_0 \beta c}\frac{1}{b}$$

- Energy transferred

$$\Delta E = \frac{\Delta p^2}{2M} = \frac{Z^2 z^2 e^4}{2M(2\pi\varepsilon_0)^2(\beta c)^2}\frac{1}{b^2}$$

Bethe-Bloch formula

- Consider α-particle scattering off Atom
  - Mass of nucleus: $M=A*m_p$
  - Mass of electron: $M=m_e$
- But energy transfer is

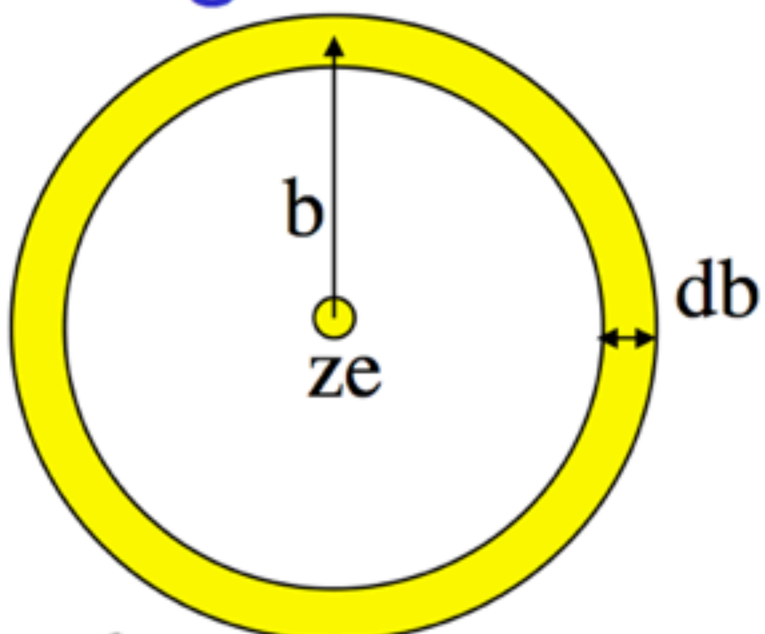
$$\Delta E = \frac{\Delta p^2}{2M} = \frac{Z^2 z^2 e^4}{2M(2\pi\varepsilon_0)^2(\beta c)^2}\frac{1}{b^2} \propto \frac{Z^2}{M}$$

- Energy transfer to single electron is

$$E_e(b) = \Delta E = \frac{2z^2 e^4}{m_e c^2 (4\pi\varepsilon_0)^2 \beta^2}\frac{1}{b^2}$$

Bethe-Bloch formula

- Energy transfer is determined by impact parameter b
- Integration over all impact parameters



$$\frac{dn}{db} = 2\pi b \times (\text{number of electrons / unit area})$$

$$= 2\pi b \times Z \frac{N_A}{A} \rho \Delta x$$

Bethe-Bloch formula

- Calculate average energy loss

$$\overline{\Delta E} = \int_{b_{\min}}^{b_{\max}} \mathrm{d}b \frac{\mathrm{d}n}{\mathrm{d}b} E_e(b) = 2C \frac{m_e c^2}{\beta^2} \frac{Zz^2}{A} \rho \Delta x \left[\ln b \right]_{b_{\min}}^{b_{\max}}$$

$$= C \frac{m_e c^2}{\beta^2} \frac{Zz^2}{A} \rho \Delta x \left[\ln E \right]_{E_{\min}}^{E_{\max}}$$

$$\text{with } C = 2\pi N_A \left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2} \right)$$

- There must be limit for E_{min} and E_{max}
  - All the physics and material dependence is in the calculation of this quantities

Bethe-Bloch formula

- Simple approximations for
  - From relativistic kinematics

$$E_{\max} = \frac{2\gamma^2\beta^2 m_e c^2}{1 + 2\gamma\frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2} \approx 2\gamma^2\beta^2 m_e c^2$$

  - Inelastic collision

$$E_{\min} = I_0 \equiv \text{ average ionisation energy}$$

- Results in the following expression

$$\frac{\overline{\Delta E}}{\Delta x} \approx 2C\frac{m_e c^2}{\beta^2}\frac{Zz^2}{A}\rho \ln\left(\frac{2\gamma^2\beta^2 m_e c^2}{I_0}\right)$$

Bethe-Bloch formula

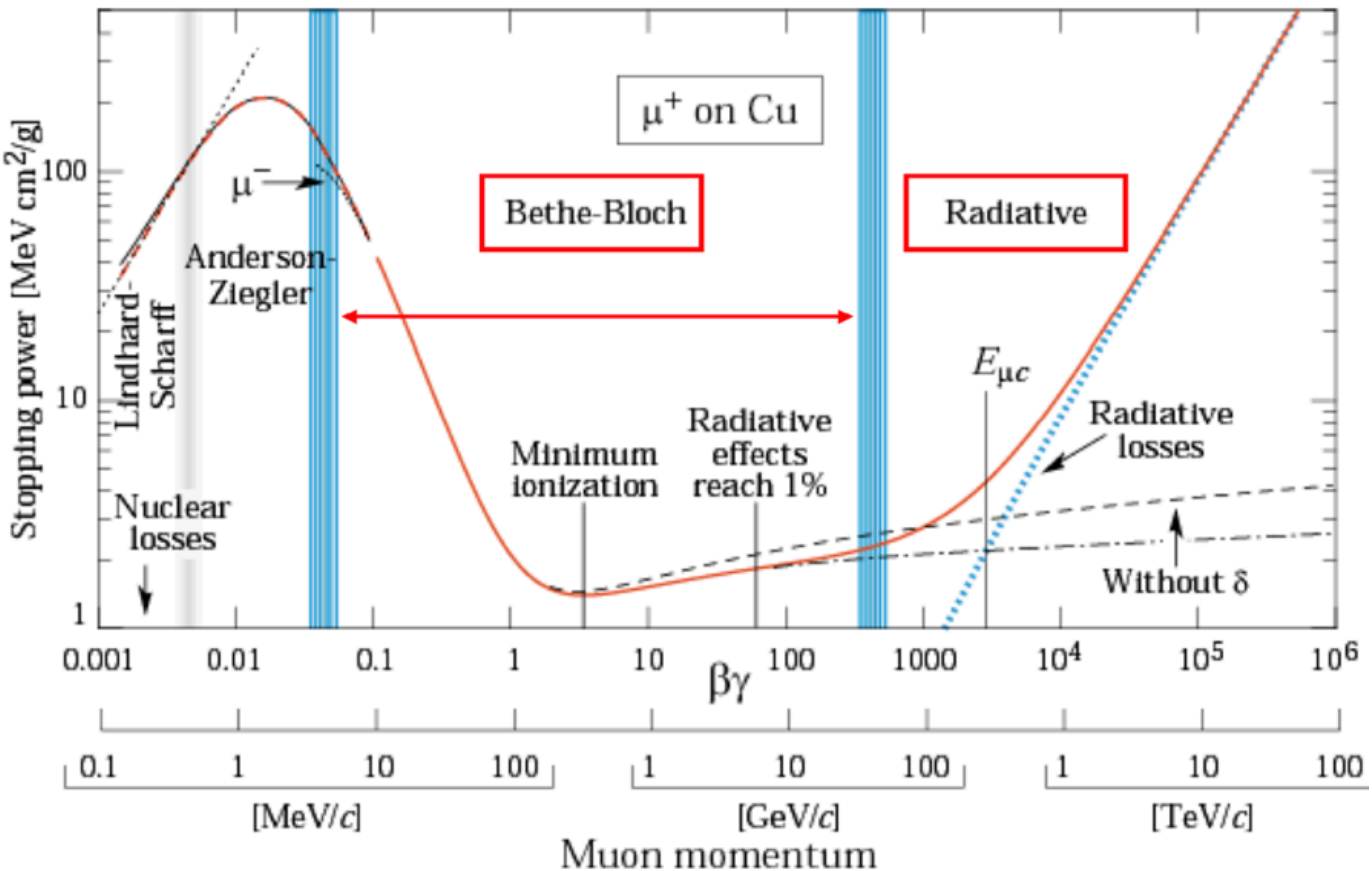
- This was just a simplified derivation
  - Incomplete
  - Just to get an idea how it is done
- The (approximated) true answer is

$$\frac{\overline{\Delta E}}{\Delta x} = 2C \frac{m_e c^2}{\beta^2} \frac{Zz^2}{A} \rho \left[\frac{1}{2} \ln \left(\frac{2\gamma^2 \beta^2 m_e c^2 E_{\max}}{I_0^2} \right) - \beta^2 - \frac{\varepsilon}{2} - \frac{\delta(\beta)}{2} \right]$$

with

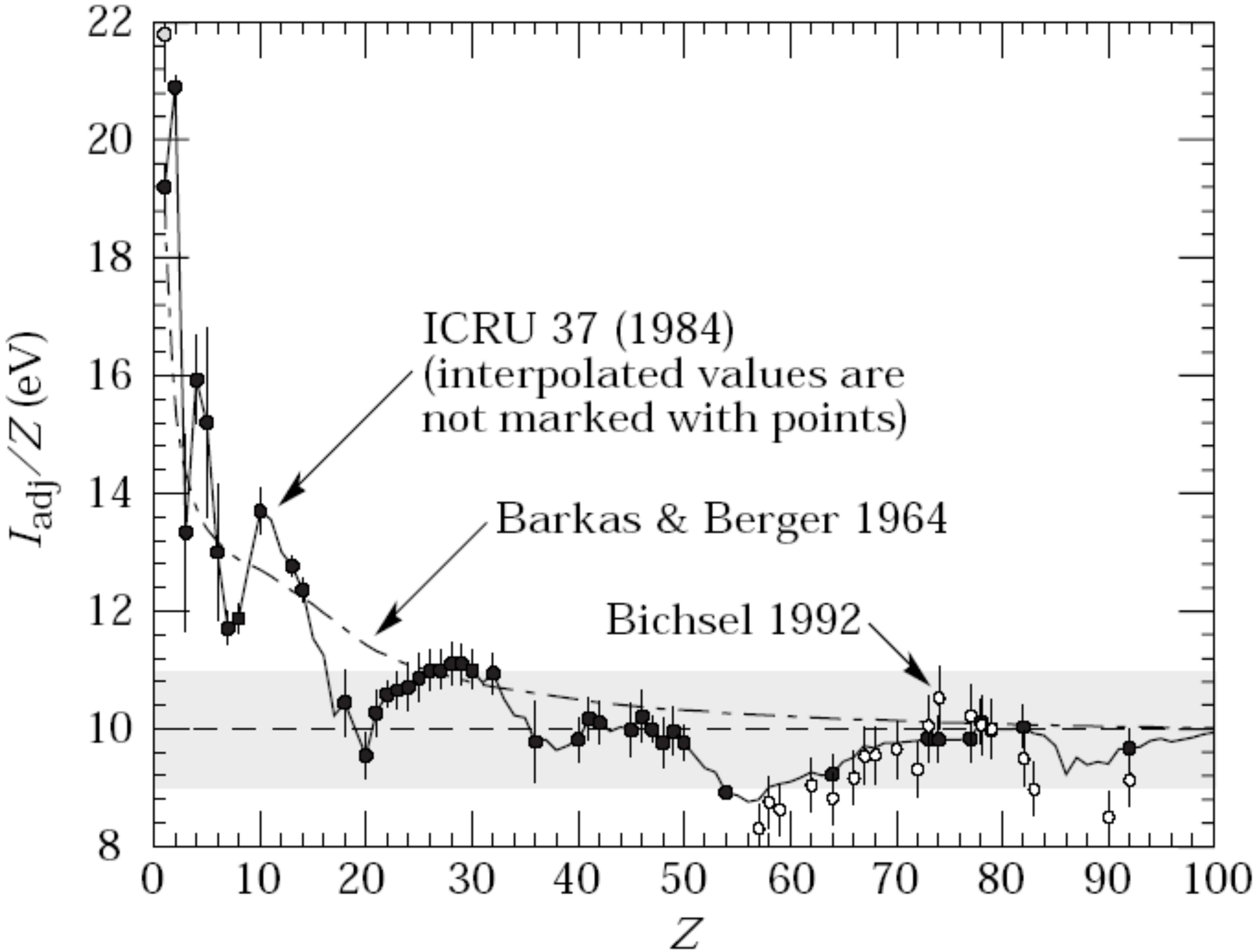
- ε screening correction of inner electrons
- δ density correction, because of polarisation in medium

Bethe-Bloch



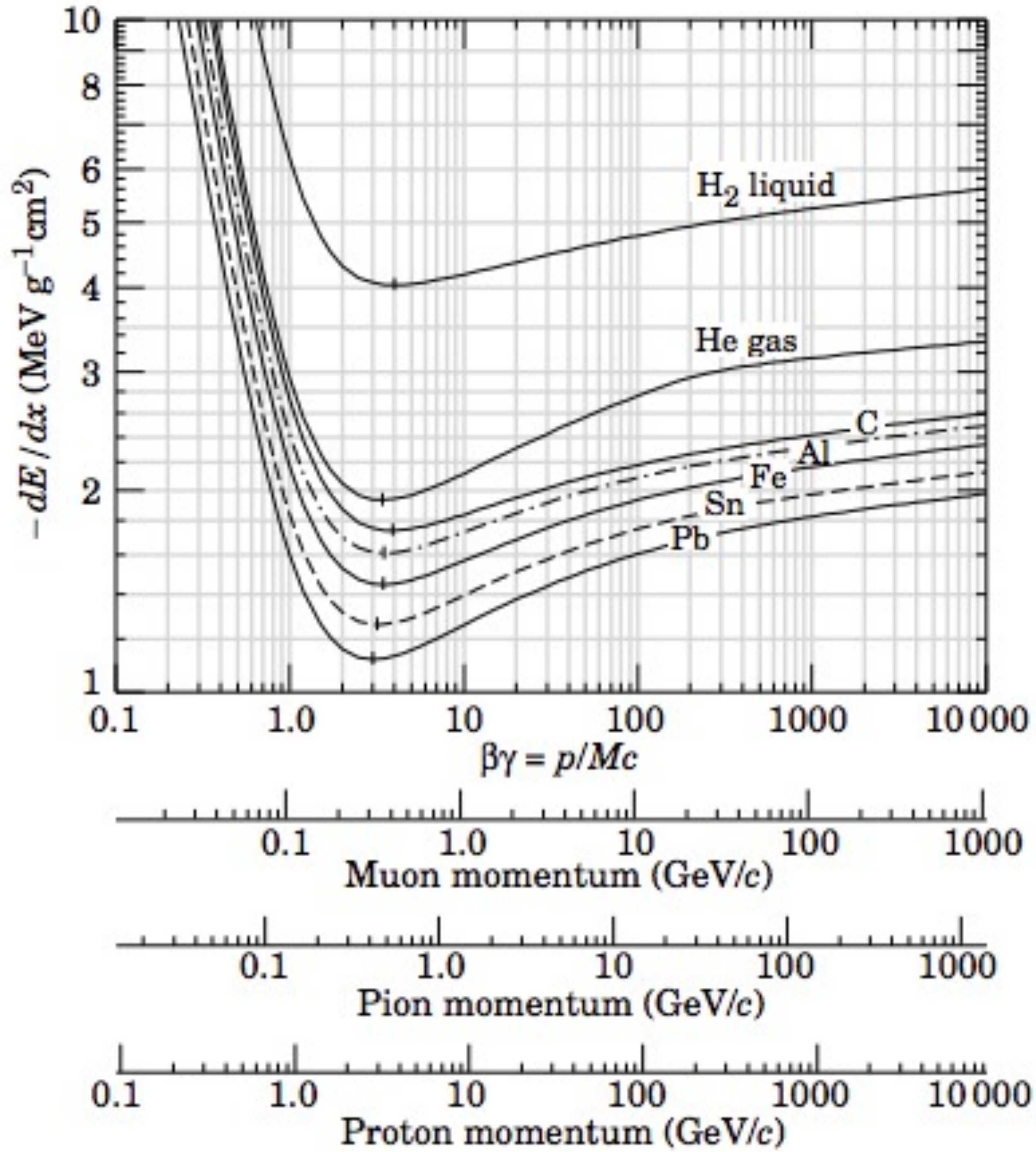
μ+ on Cu
Bethe-Bloch
Radiative
μ−
Anderson-
Ziegler
Lindhard-
Scharff
Nuclear
losses
Minimum
ionization
Radiative
effects
reach 1%
Eμc
Radiative
losses
Without δ
Stopping power [MeV cm2/g]
100
10
1
0.001
0.01
0.1
1
10
100
1000
10^4
10^5
10^6
βγ
0.1
1
10
100
1
10
100
1
10
100
[MeV/c]
[GeV/c]
[TeV/c]
Muon momentum

Ionization constant



I/Z=10 except for low Z elements

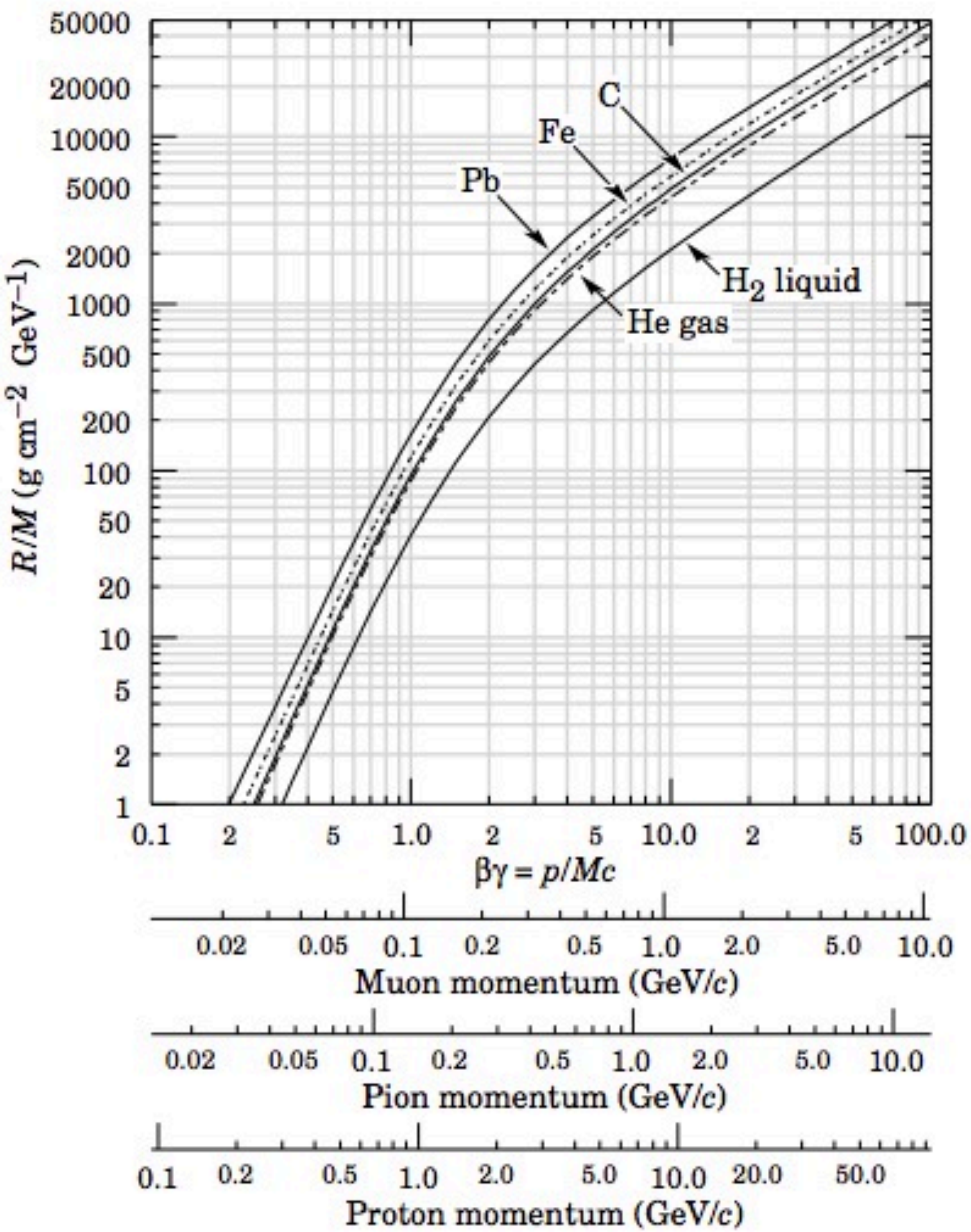
Different materials



Minimum ionizing particle (mip) energy is the same whatever the material

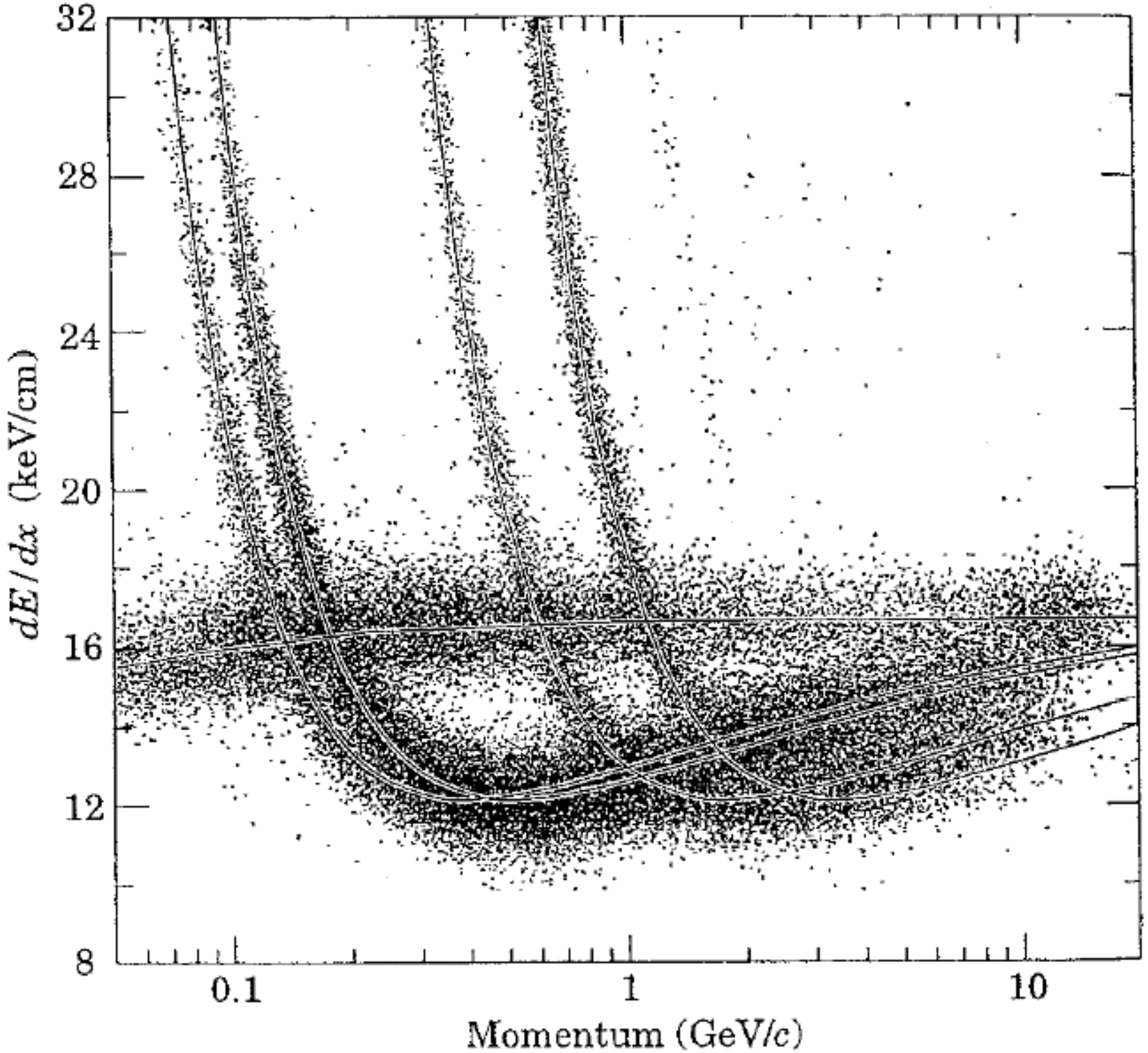
$dE/dx = 2\text{MeV g}^{-1}\text{ cm}^2$

Range



Example :
K^+ 700MeV in Lead
1) p/Mc = 1.42
2) R/M=400
3) R=197 g/cm^2
4) D=17cm

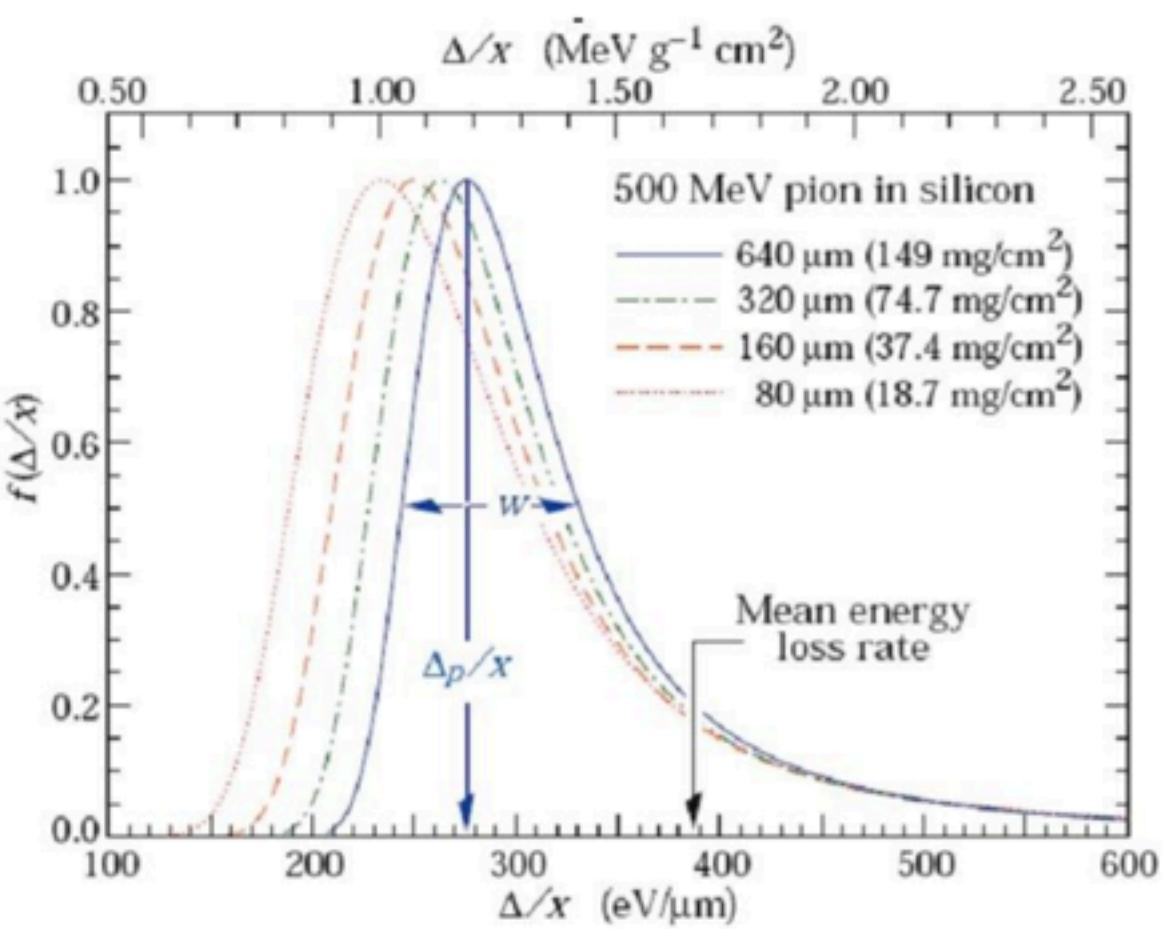
dE/dx for particle identification

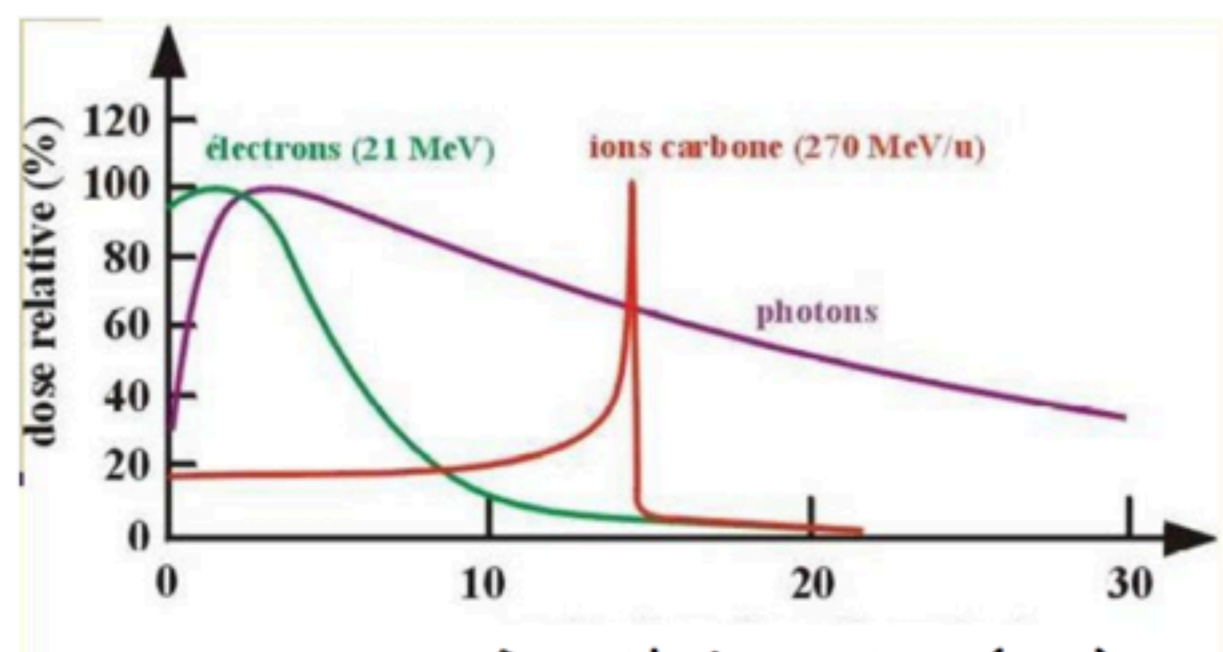


dE/dx remarks

Bethe Bloch describes the average energy loss. For moderate thickness absorber fluctuations on this energy loss described by a Landau distribution
For thin absorber (small dx) fluctuations become large

The energy loss is larger at small E (energy), i.e. end of the path in matter
→Bragg peak
Not used in HEP but is basic for medical application, hadron therapy





dE/dx illustrative numbers

Energy loss of a 10 GeV muon in 1 cm of plastic scintillator (γ = 1) or a gas chamber (γ = 0.001) ?

Muons can be considered as a MIP with 2 MeV/(g/cm^2)
→ 2 MeV in 1 cm scintillator
→ 2 keV in 1 cm of gas
To stop a 450 GeV muon beam, will need 900 m of concrete (density 2.5) !

How many meters of air to stop an α particle of 2 MeV ?

Particle with very low β (below the minimum ionization)
dE/dx around 700 MeV /(g/cm^2) and ρ =1g/l → 0.7 MeV/cm
Can stop α in 2-3 cm of air

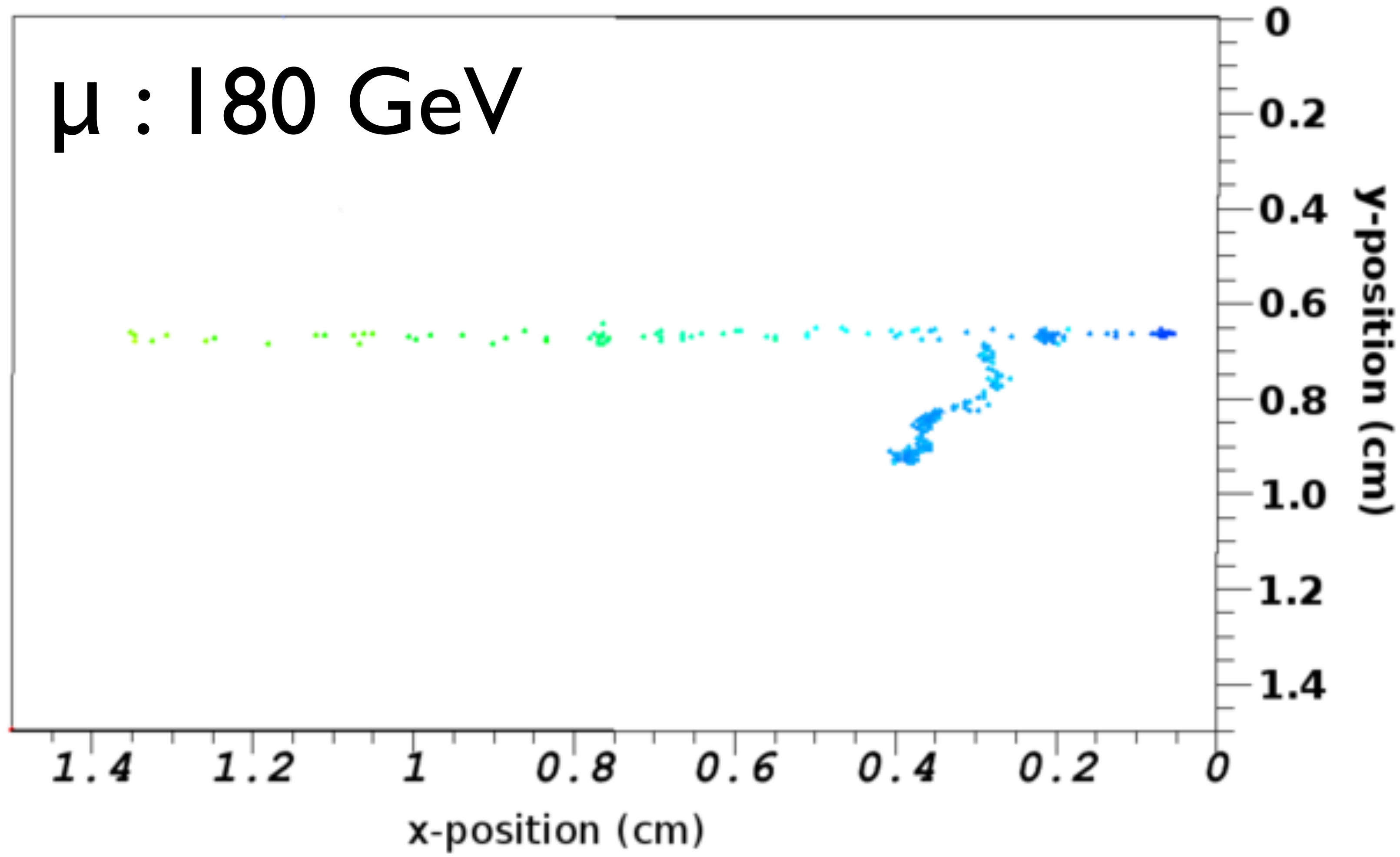
δ rays

- Energy loss distribution is not Gaussian around mean.
- In rare cases a lot of energy is transferred to a single electron

δ-Ray

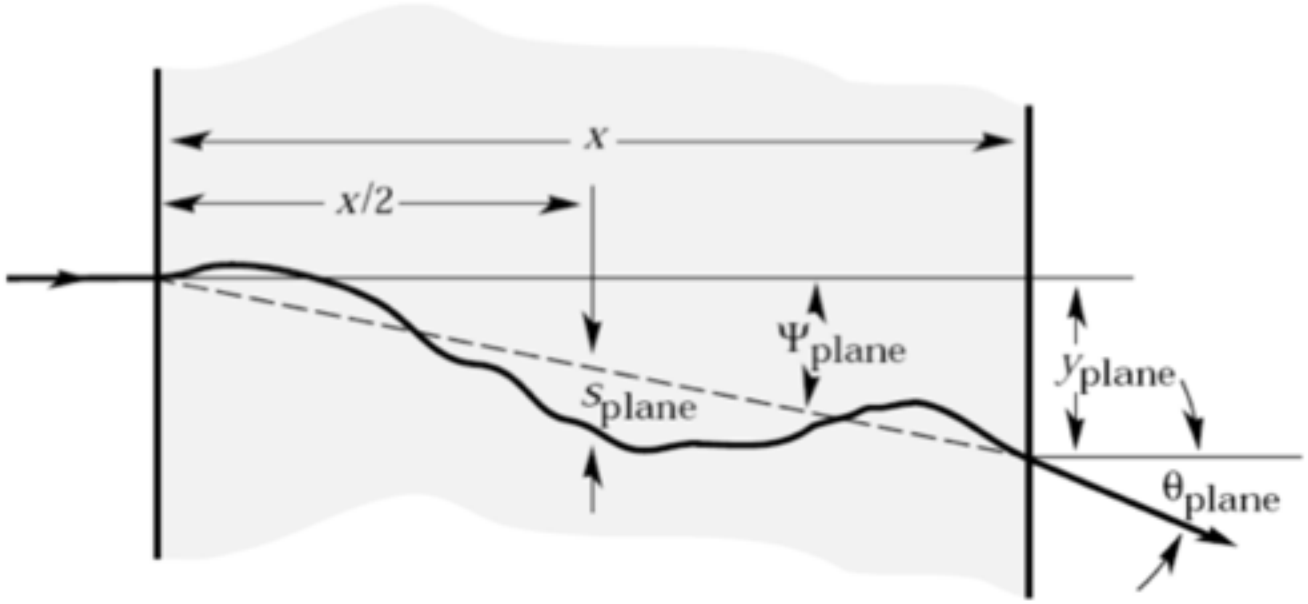
I<<E<T_{max}

$$T_{\max} = \frac{2m_e c^2\,\beta^2\gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

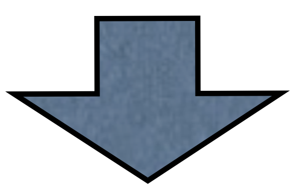


Multiple scattering

- Particles don't only loose energy …



… they also change direction



Most of this deflection is due to Coulomb scattering from nuclei, and hence the effect is called multiple Coulomb scattering

- Average scattering angle is roughly Gaussian for small deflection angles
- With

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln\left(\frac{x}{X_0}\right)\right]$$

$$X_0 \equiv \text{radiation length}$$

- Angular distributions are given by

$$\frac{dN}{d\Omega} \propto \frac{1}{2\pi\theta_0^2} \exp\left(-\frac{\theta_{space}^2}{2\theta_0^2}\right)$$

$$\frac{dN}{d\theta_{plane}} \propto \frac{1}{\sqrt{2\pi}\theta_0} \exp\left(-\frac{\theta_{plane}^2}{2\theta_0^2}\right)$$

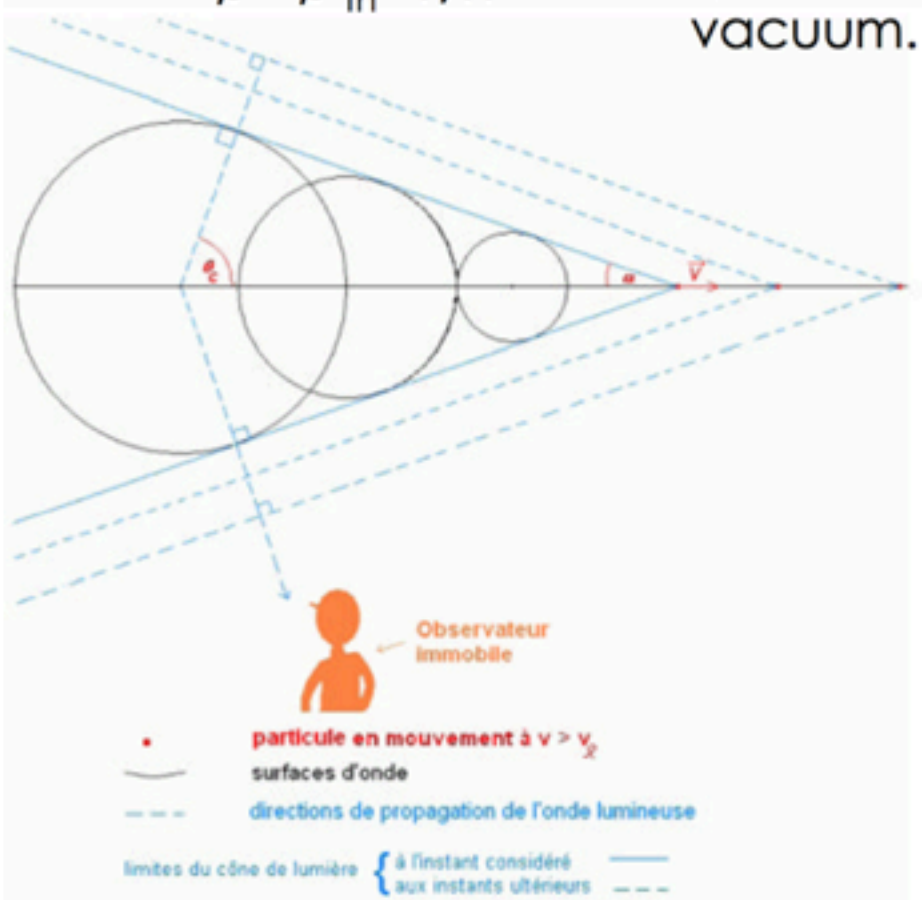
Cherenkov radiation

Cherenkov radiation arises when a charged particle in a material medium moves faster than the speed of light in the same medium:

$$\boxed{v_{particle} > c/n}$$

where n is the index of refraction and c the speed of light in vacuum.

$\beta \geq \beta_{th} = 1/n$



An electromagnetic shock wave is created and the coherent wavefront formed is conical and emitted at a well-defined angle:

$$\boxed{\cos\,\theta_c = 1/\beta\,n}$$

* Number of photons emitted per unit length of radiator per unit wavelength (Frank-Tamm formula):

$$\frac{d^2N}{d\lambda dx} = \frac{2\pi z^2\alpha}{\lambda^2}\left(1-\frac{1}{\beta^2 n^2}\right) = \frac{2\pi z^2\alpha}{\lambda^2}\sin^2\theta_c$$

α is the fine structure constant

z is the charge of the moving particle

Transition radiation

- A charged particle (ze) passing through media of different indices of refraction
- The continuity of the electric field at the interface leads to the creation of a radiation calls “radiation of transition”.
- Emission in the X-rays domain.
- Emission angle forward and proportionnal to $1/\gamma$ (Lorentz factor)
- If passage between vacuum and medium with plasma frequency ω_p:

$$\boxed{E_{rad.} = \tfrac{1}{3} z^2 \alpha \mathrm{h} \omega_p \gamma} \qquad \boxed{n_{photon} \propto z^2 \alpha \ln(\gamma)^2}$$

- Example: e^- (5 GeV) → Lorentz factor ~ 10^4

  → $\langle N_{photon} \rangle$ ~ 18.4 α = 13 x10^{-2}

→ More interfaces to increase the number of photons.

Electrons/positrons

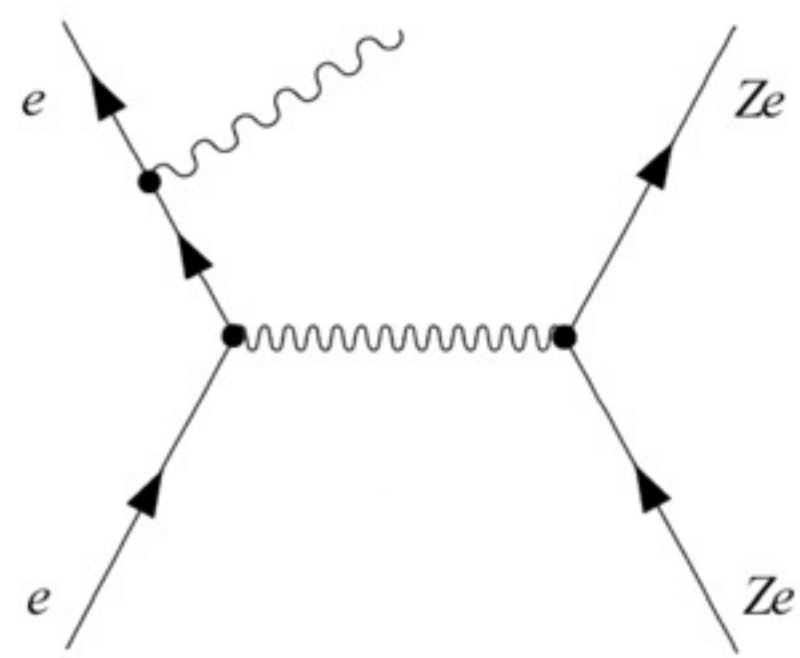
Energy loss for electrons/positrons involve mainly two different physics mechanisms:

- Excitation/ionization

  But collision between identical particles + electron is now deflected

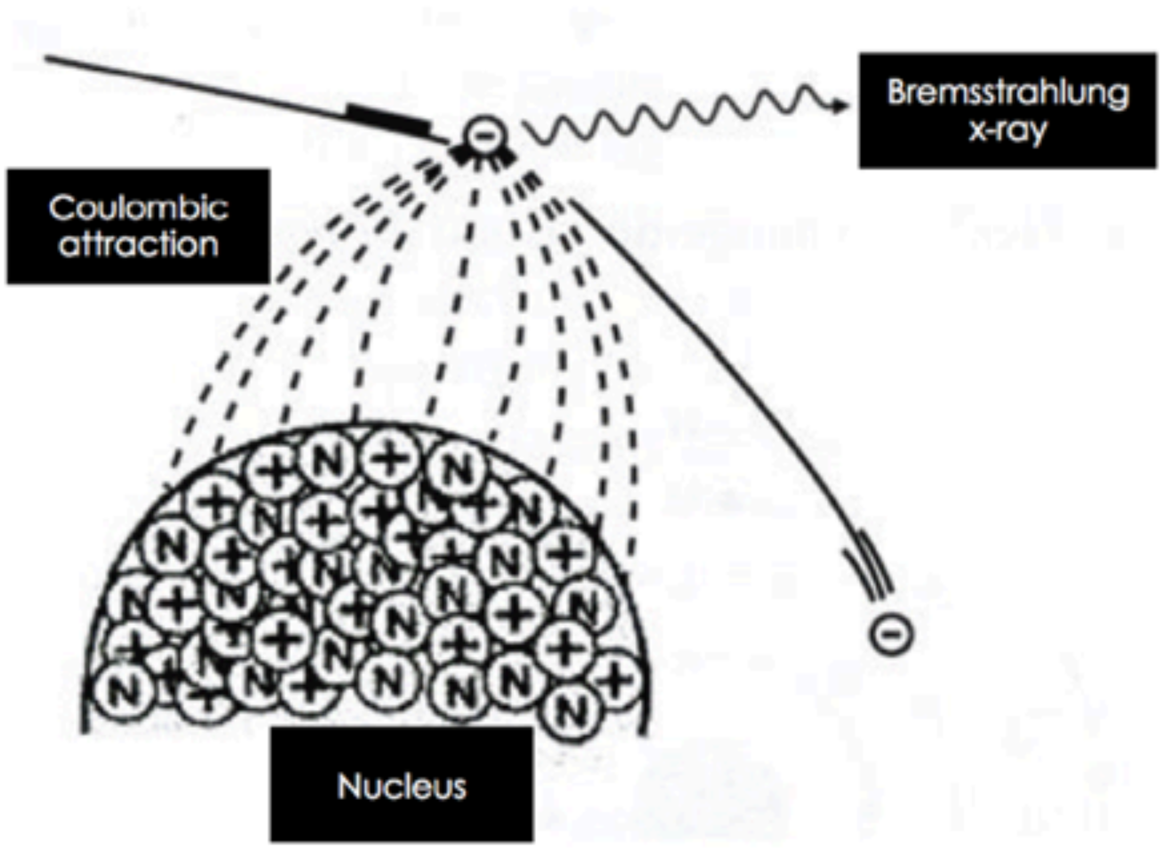
- Bremsstrahlung : emission of photon by scattering with the nucleus electrical field

  At high energies radiative processes dominate



Bremsstrahlung

deflection → acceleration → radiation



$$-\left(\frac{dE}{dx}\right)_{Brem.} = 4\alpha N_A \frac{Z^2}{A} z^2 \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mc^2}\right)^2 E \ln\frac{183}{Z^{1/3}} \propto \frac{E}{m^2}$$

m_0 : electron rest mass
E : electron energy
ρ : density of the absorber
Z : atomic number of the absorber

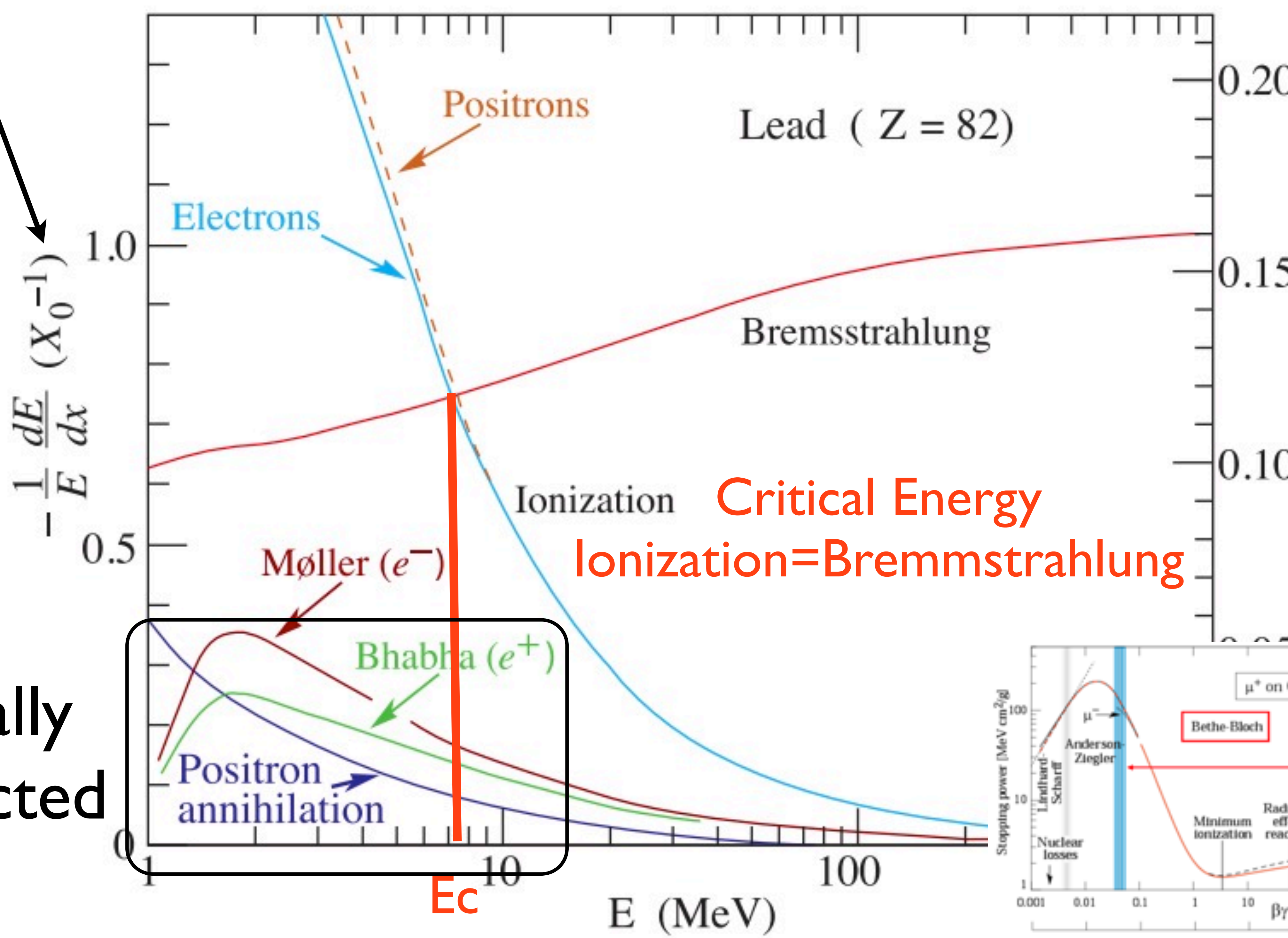
Proportional to $1/m^2$ so:
- muon radiation lower for same energy (~$[0.5/106]^2$)
- main effect for all charged particles if E > TeV

Radiative losses are most important for:
- high electron energy
- absorber material of large atomic number

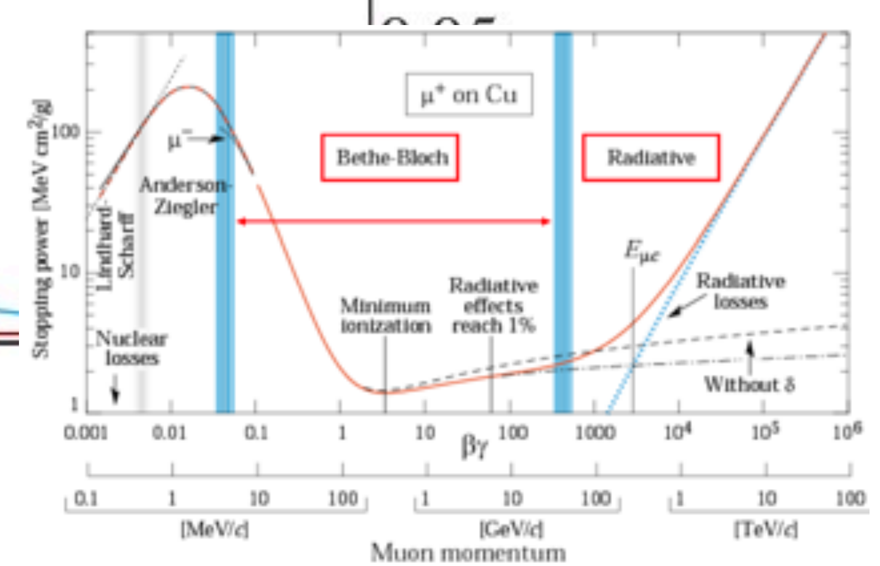
Photon: low energy and in the direction of the incident electron

Electron interaction with matter

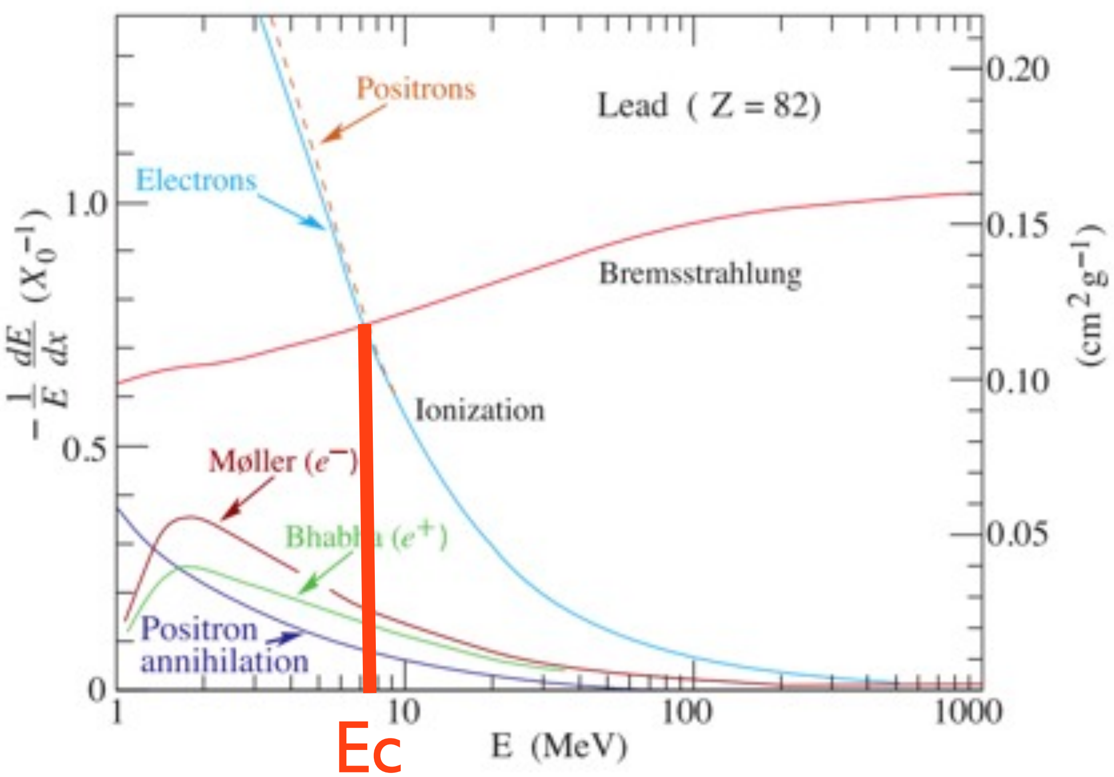
X_0 : Radiation length

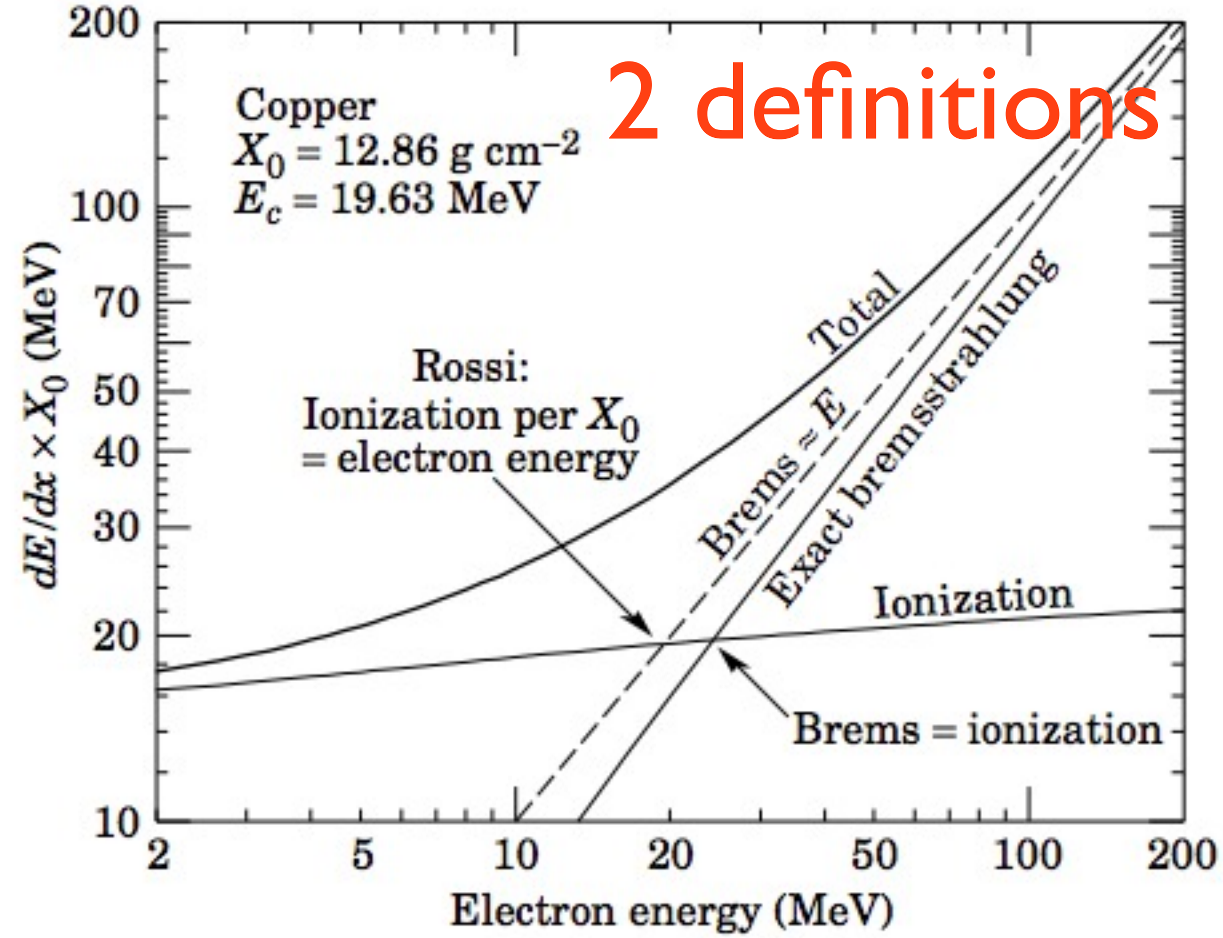


Usually neglected

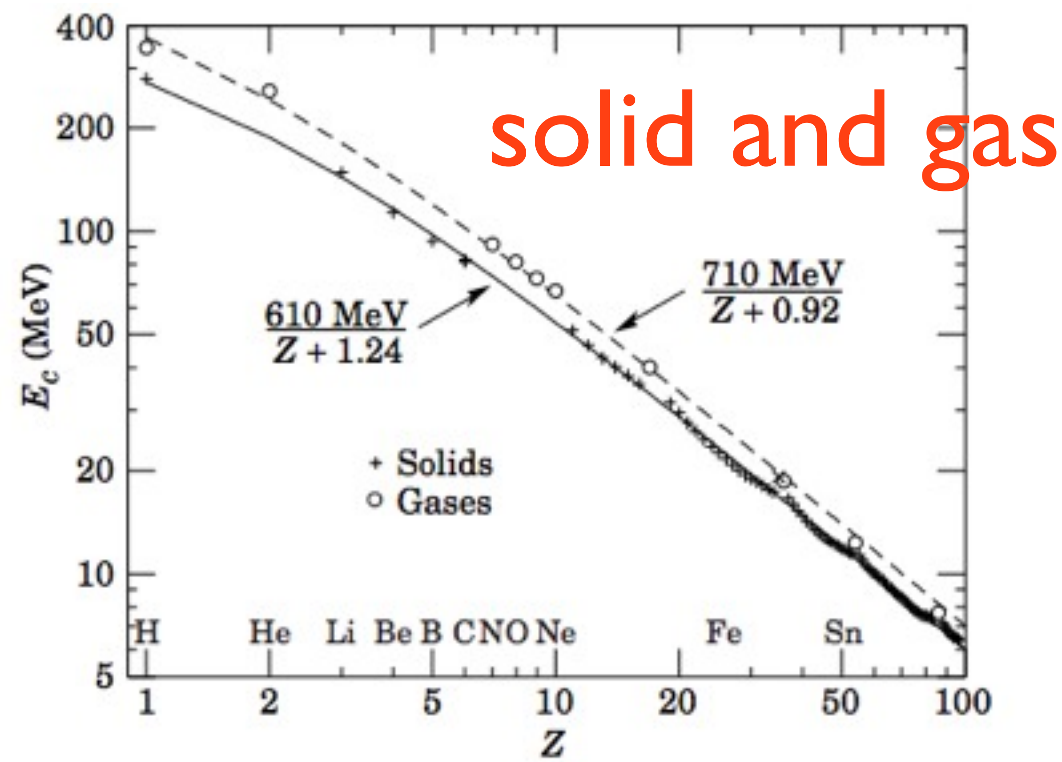


Critical Energy





Material	Critical energy [MeV]
Pb	9.51
Al	51.0
Fe	27.4
Cu	24.8
Air (STP)	102
Lucite	100
Polystyrene	109
NaI	17.4
Anthracene	105
H_2O	92



Radiation length

$$E(x) = E_0 \exp(-x/X_0) \quad \rightarrow \quad E(X_0) = E_0/e$$

X_0 : distance over which the electron energy is reduced by a factor $1/e$ due to radiation loss only.

X_0 is often expressed in surface density (g.cm^{-2})

so X_0[g.cm^{-2}] = ρX_0 (cm)

$$X_0\left[g \cdot cm^{-2}\right] = \frac{716.4 \cdot A}{Z(Z+1)\ln\left(287/\sqrt{Z}\right)}$$

For compound material:

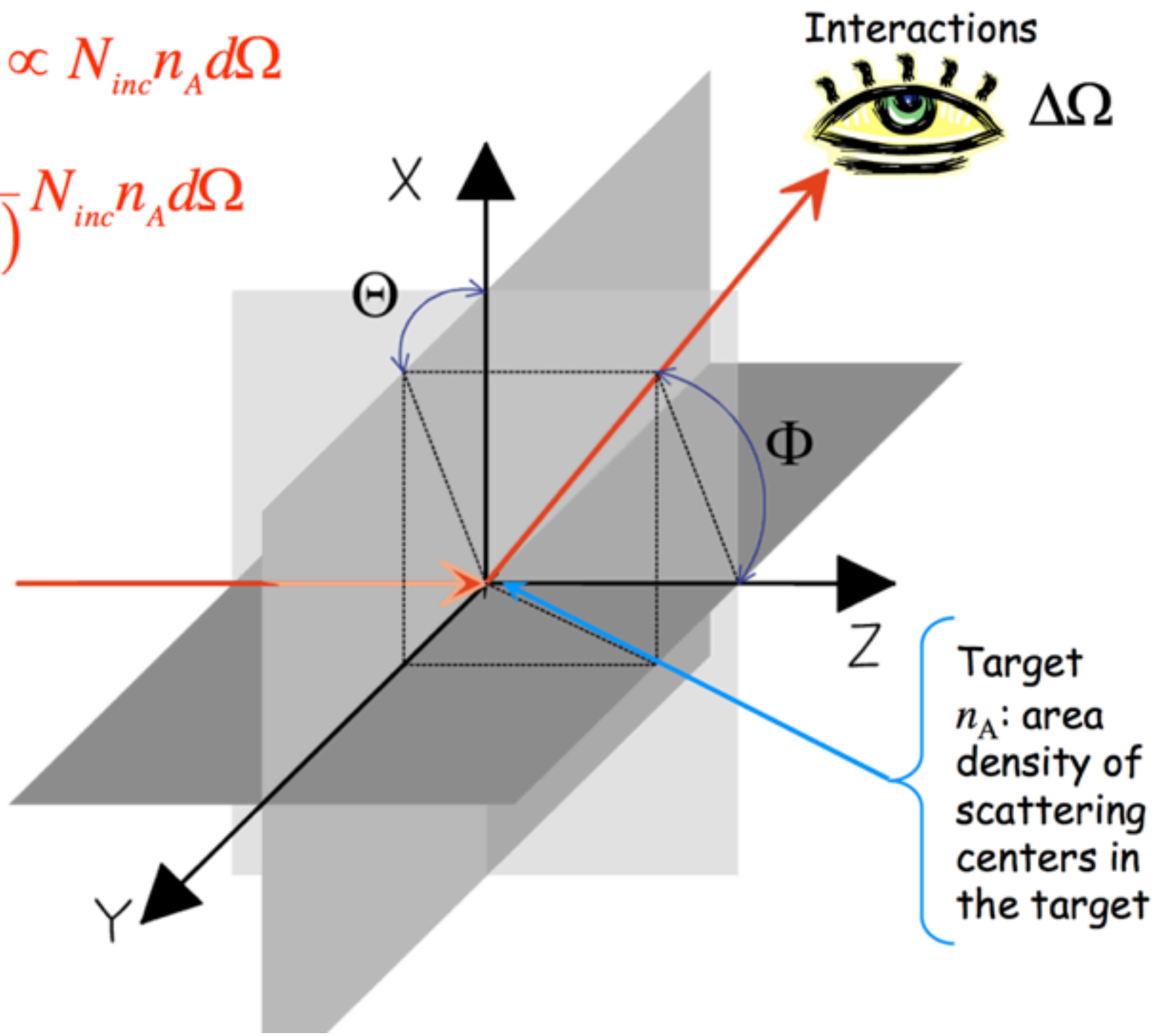
$1/X_0 = w_1[1/X_0]_1 + w_2[1/X_0]_2 + \ldots$

With w_1, w_2, ... fraction by weight of each material.

Material	[gm/cm^2]	[cm]
Air	36.20	30050
H_2O	36.08	36.1
NaI	9.49	2.59
Polystyrene	43.80	42.9
Pb	6.37	0.56
Cu	12.86	1.43
Al	24.01	8.9
Fe	13.84	1.76
BGO	7.98	1.12
BaF_2	9.91	2.05
Scint.	43.8	42.4

Cross section

$$N_{scat}(\Theta,\Phi) \propto N_{inc} n_A d\Omega$$
$$= \frac{d\sigma}{d\Omega(\Theta,\Phi)} N_{inc} n_A d\Omega$$



The cross section is related to the probability of a given phenomena to occur (measured in barns : 1 barn = 10^{-24} cm^2)

Photon Interaction

Photons will interact with:

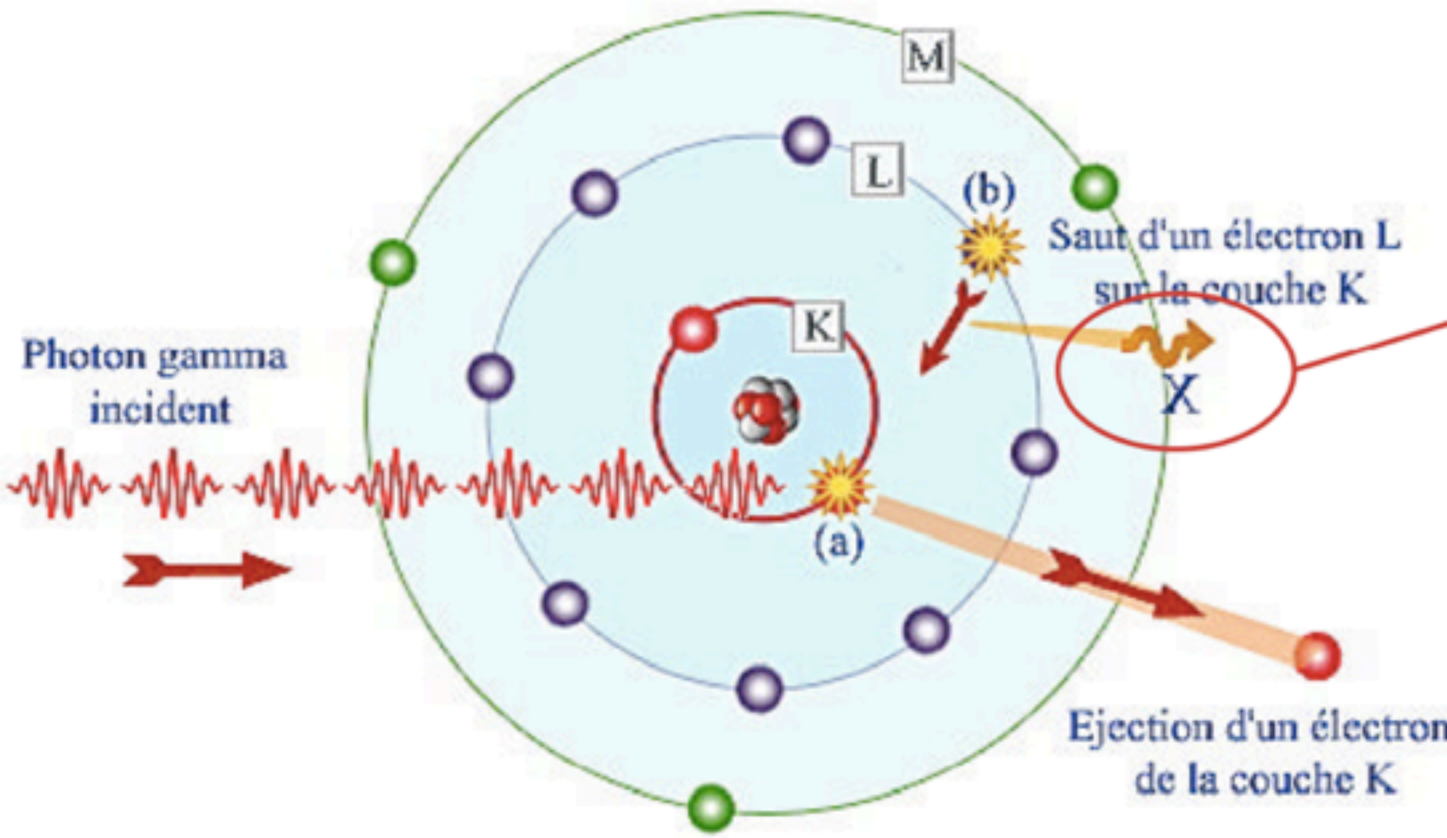
- Atomic electrons
- Nucleus
- Electromagnetic field (electrons or nucleus)

With:

- No energy loss (elastic diffusion) →→ Thomson-Rayleigh
- Partial energy loss (inelastic diffusion) →→ Compton effect
- Total energy loss (absorption) →→ photo-electrique effect

→→ pair production

Photon electric effect



Internal rearrangement leads to the emission of a X photon.

If the rearrangement leads to the emission of one or more electron
→ Auger effect

$$\gamma + {}^{A}X \rightarrow {}^{A}X^{*} + e^{-}$$

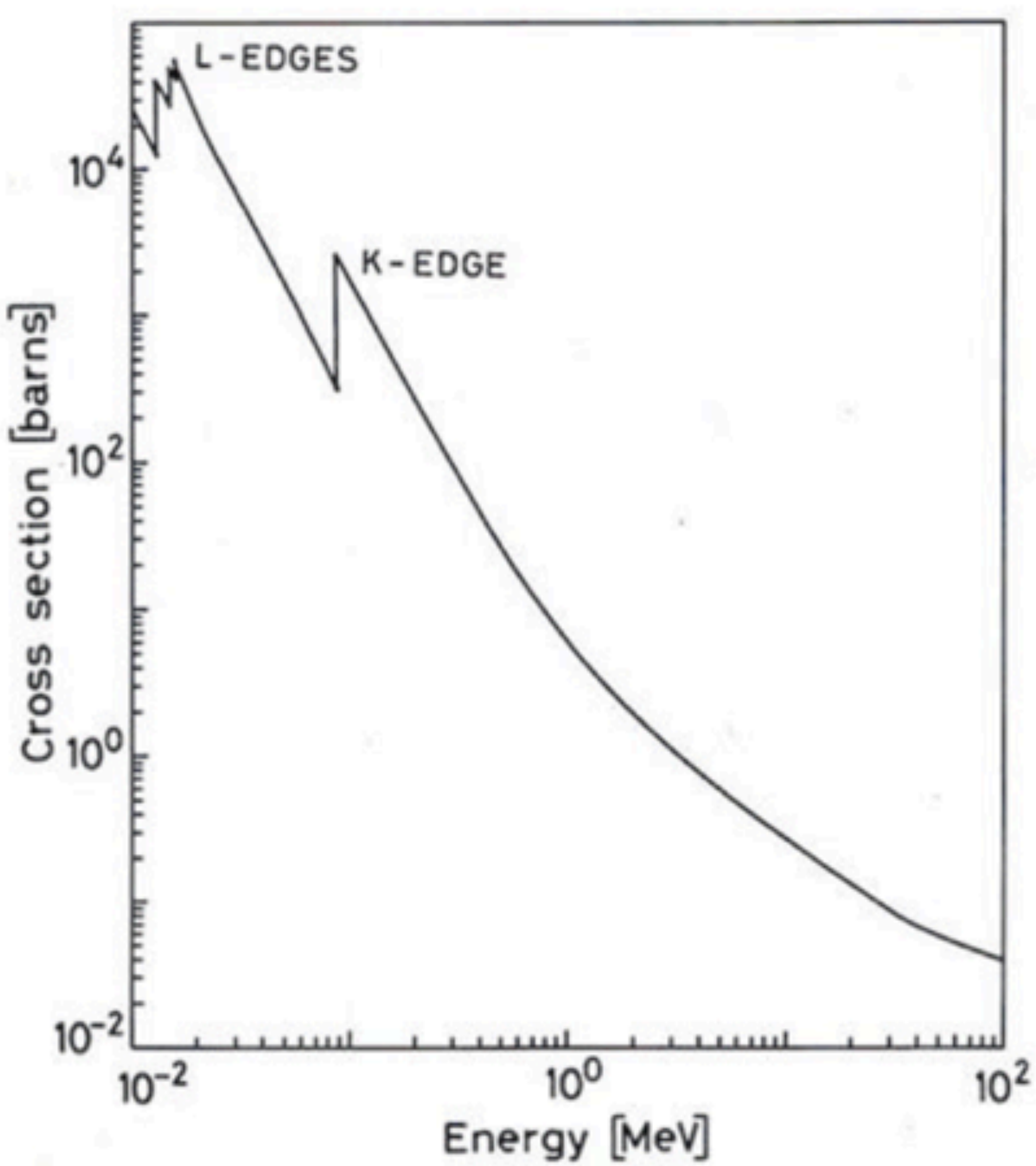
Energy of the outgoing electron: $E = E_{\gamma} - E_b$

where $E_{\gamma} = h\,\nu$

and E_b is the binding energy of the electron

The nucleus is absorbing the recoil momentum

Photon electric cross section



Calculated photoelectric cross section for lead

Photoelectric effect is:

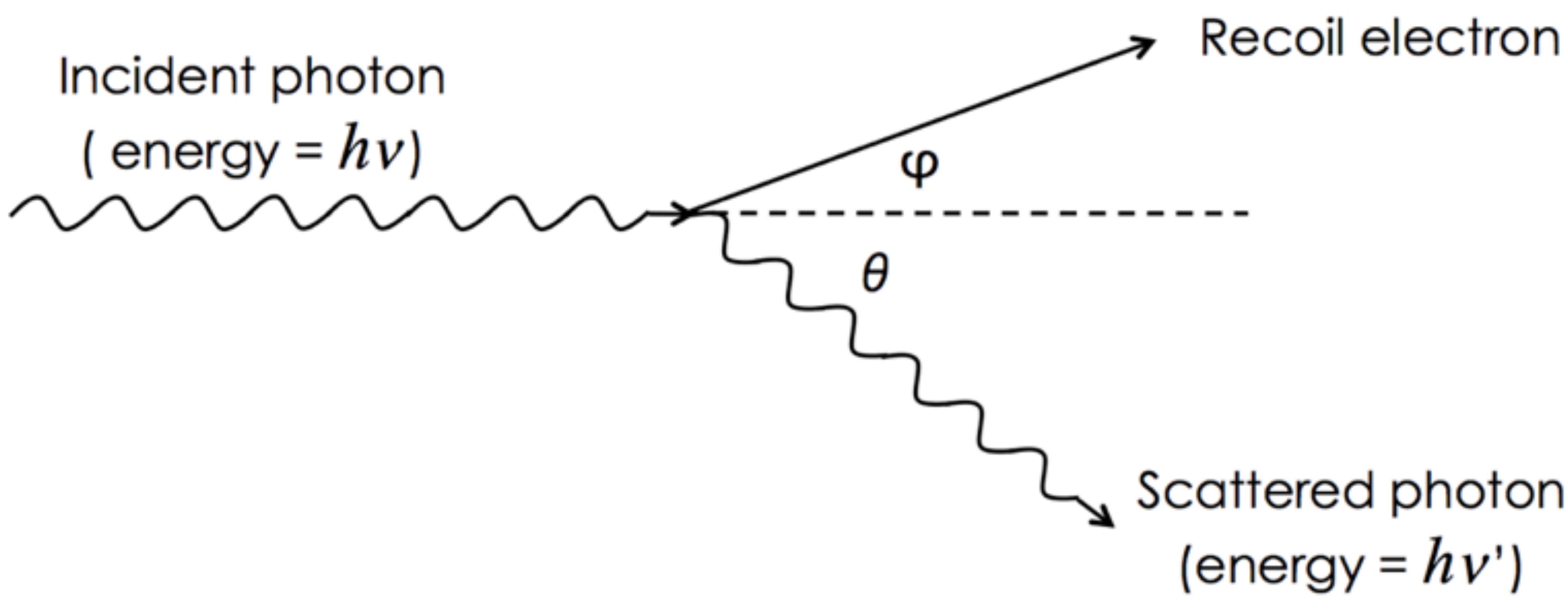
- possible for $E_\gamma > E_b$
- maximum for $E_\gamma = E_b$

$$\tau \cong C \times \frac{Z^n}{E_\gamma^{3.5}}$$

with C a constant and n between 4 and 5 depending on the photon energy.

- τ very high at low energy
- Shielding with high Z materials

Compton scattering

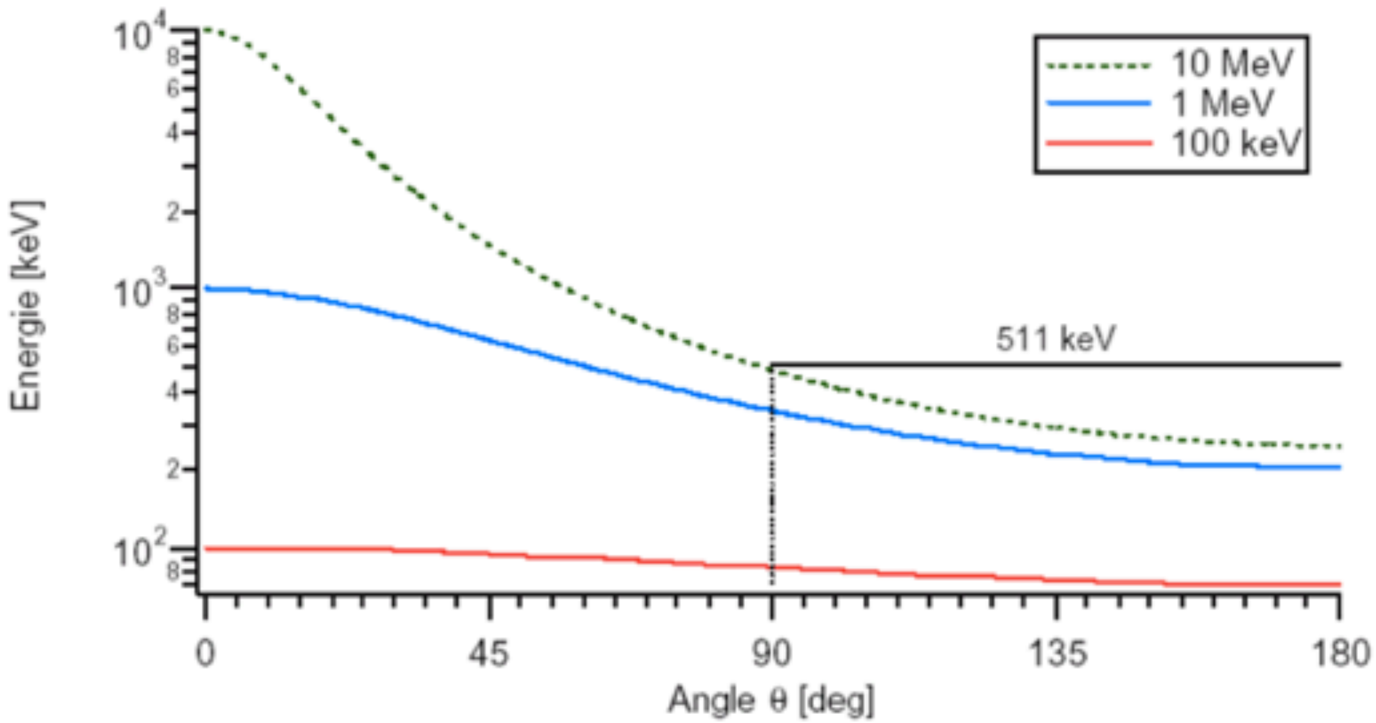


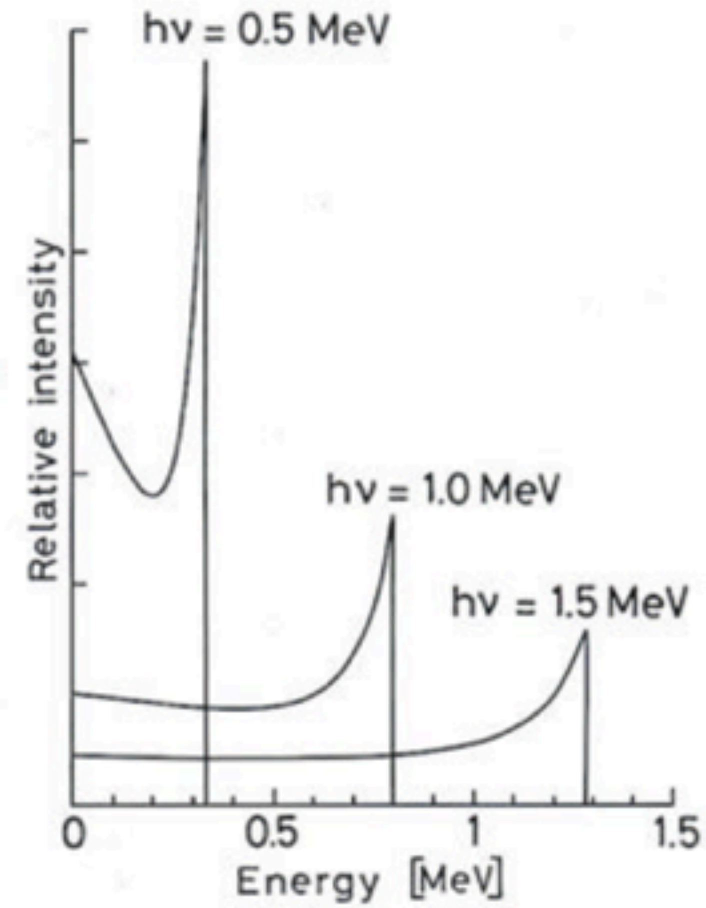
$$\boxed{\gamma + {}^{A}X \rightarrow {}^{A}X^{*} + e^{-} + \gamma'}$$

$$h\nu' = \frac{h\nu}{1+\alpha(1-\cos\theta)} \qquad E_{e^-} = h\nu\frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}$$

with $\alpha = h\nu/m_e c^2$ (m_e: mass of the electron)

Electron and photon energy





* Scattered photon energy as a function of the angle for 3 energies of incident photon
* For θ> 90°, the energy of the scattered photon is always lower than 511 keV
* Energy distribution of Compton recoil electron (compton edge)

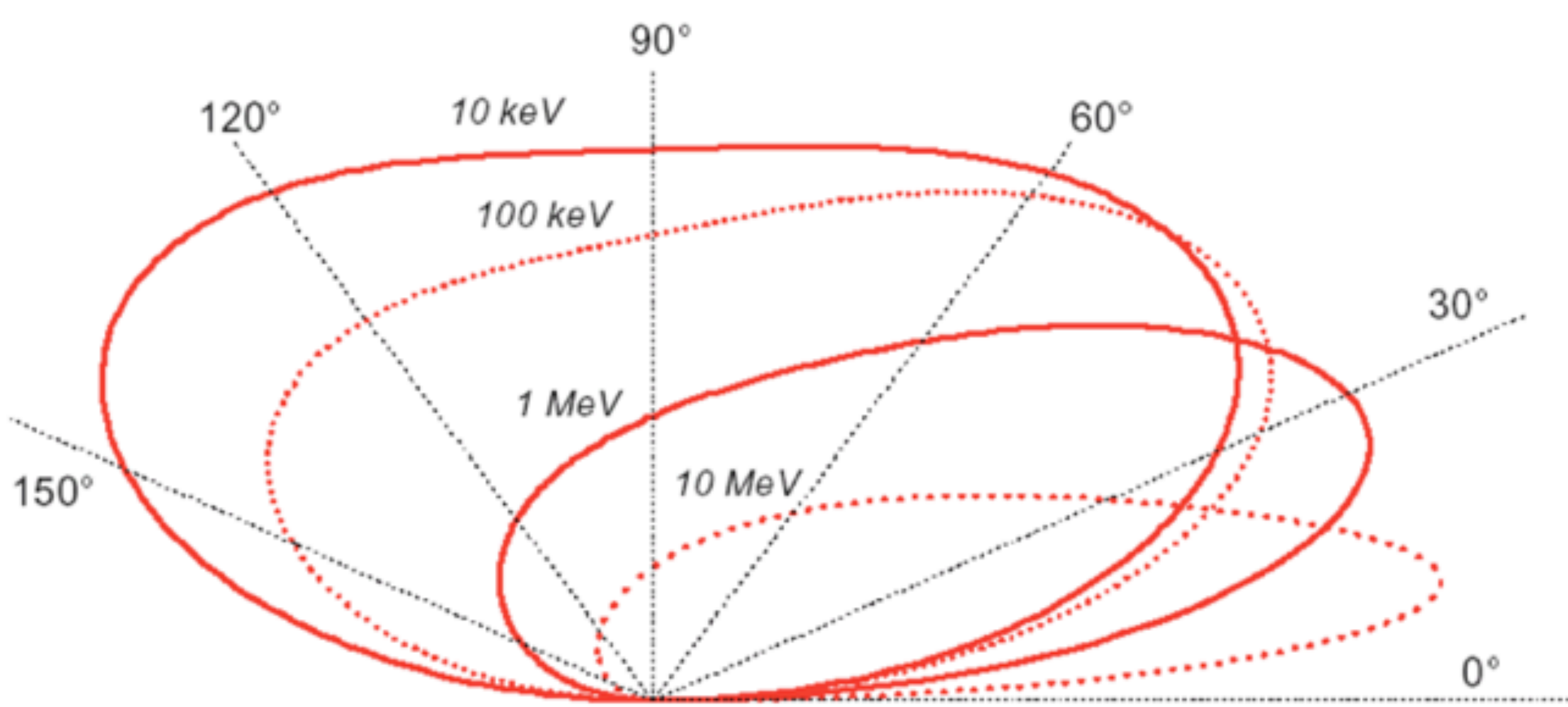
$$E_{e^-}^{\max} = \frac{h\nu}{\frac{m_e c^2}{2h\nu} + 1}$$

Angular distribution

* Quantum electrodynamics (Klein and Nishina – 1929)
  → differential scattering cross section:

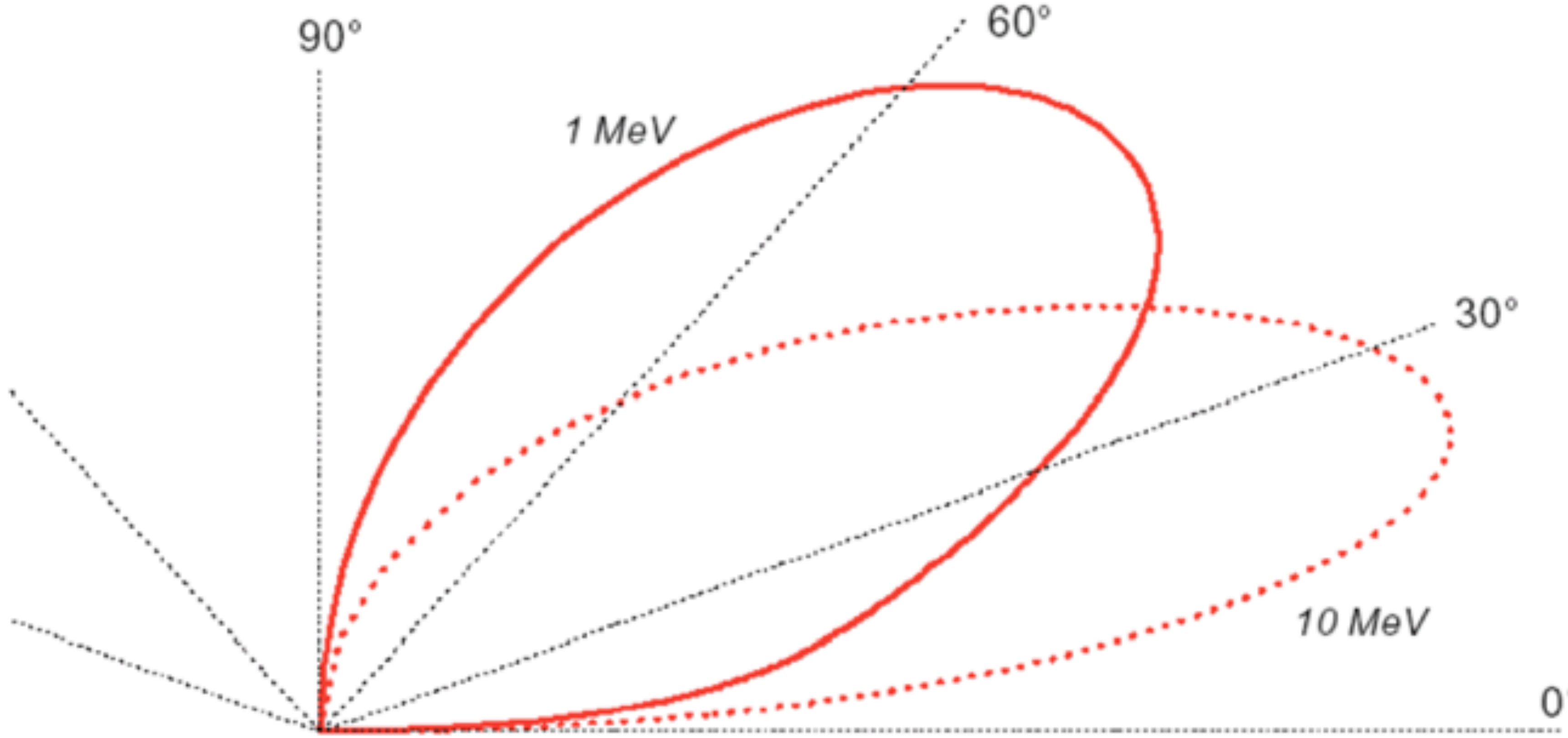
$$\frac{d\sigma_{KN}(h\nu,\theta)}{d\Omega}=\frac{r_e^2}{2}\left(\frac{1+\cos^2\theta}{\left(1+\alpha(1-\cos\theta)\right)^2}+\frac{\alpha^2(1-\cos\theta)^2}{\left(1+\alpha(1-\cos\theta)\right)^3}\right)$$

where r_e is the classical electron radius $r_e=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_0c^2}=2{,}818\times10^{-15}\,\mathrm{m}$



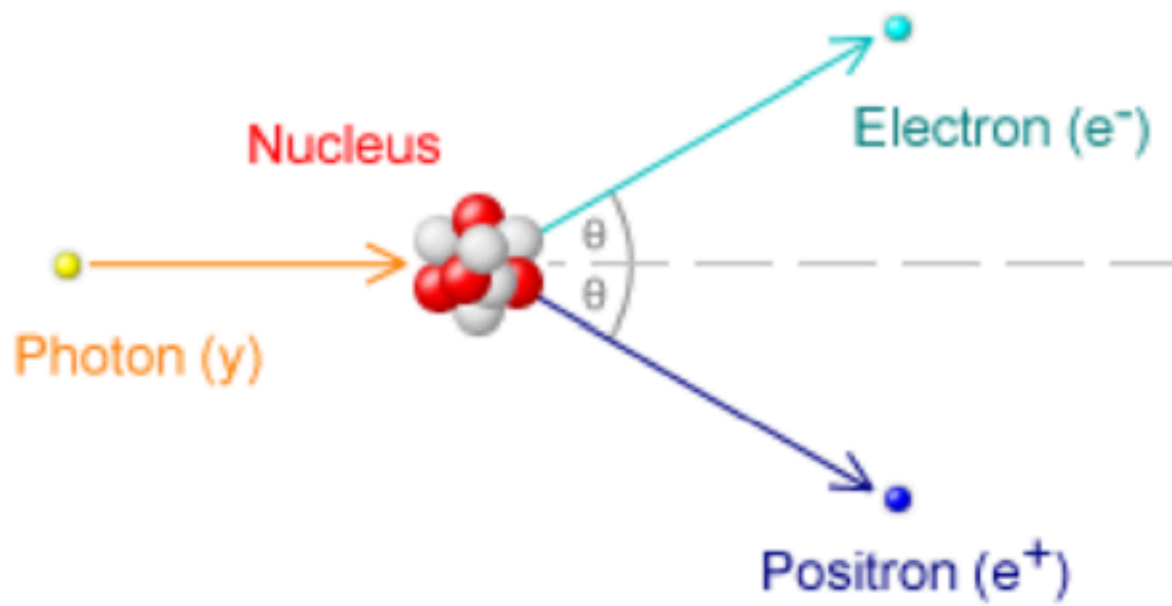
Angular distribution of the scattered photon

Electron angular distribution



- Electron compton: forward distribution
- K-N equation → free electron hypothesis
  - Ok for electrons on external layers
  - Corrections for deeper electrons, specially at low energy and high Z

Pair production



$$\boxed{\gamma + {}^{A}X \rightarrow {}^{A}X^{*} + e^{-} + e^{+}}$$

$$E_{\min} = 2m_0c^2\left(1 + \frac{m_0}{M}\right)$$

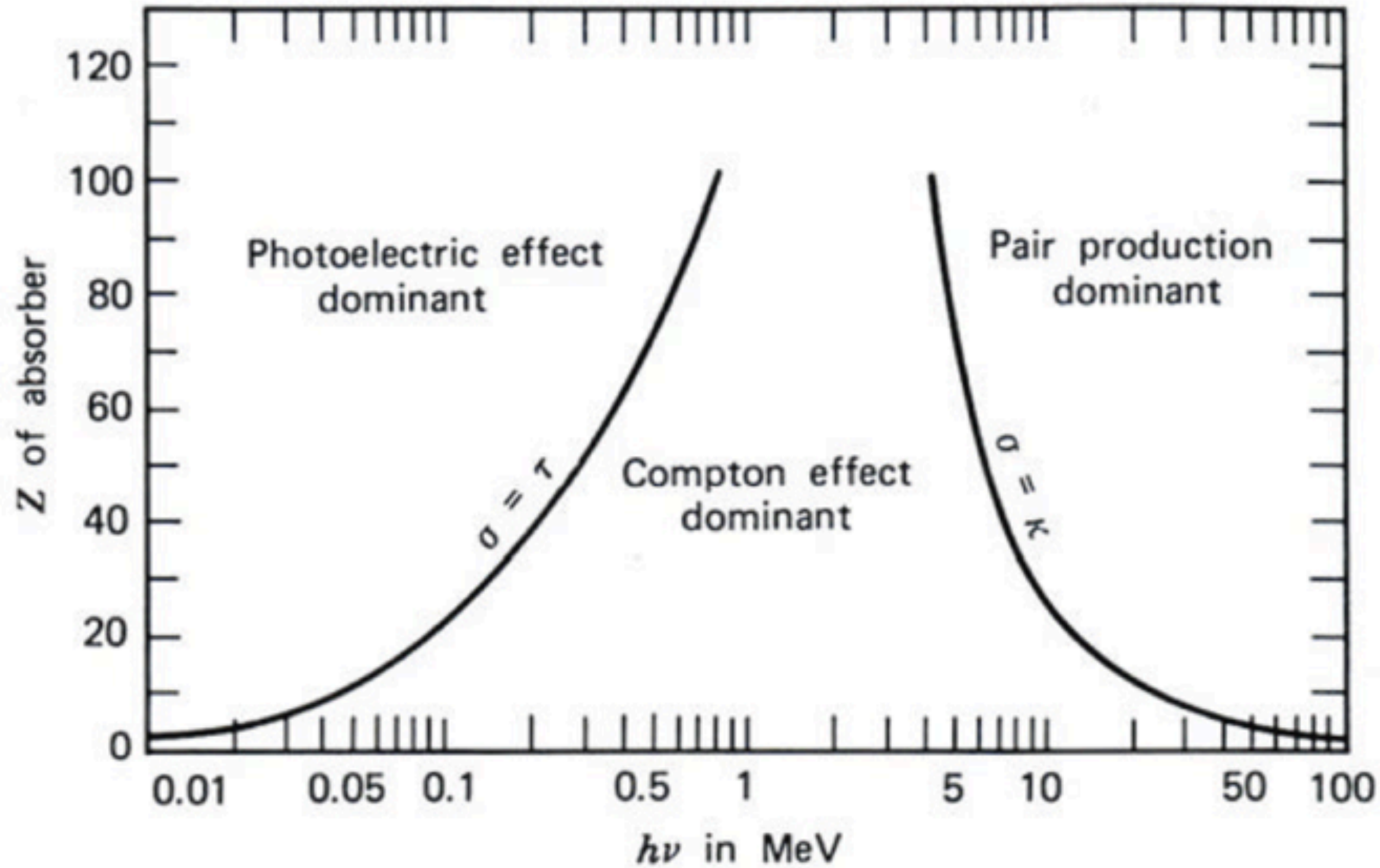
with M the mass of the nucleus (or atomic electron)

* <u>Annihilation electron-positron</u>

  The positron will loose its kinetic energy by multiple scattering (few millimeters), meet an electron of the media and 2 photons of 511 keV are emitted back-to-back.

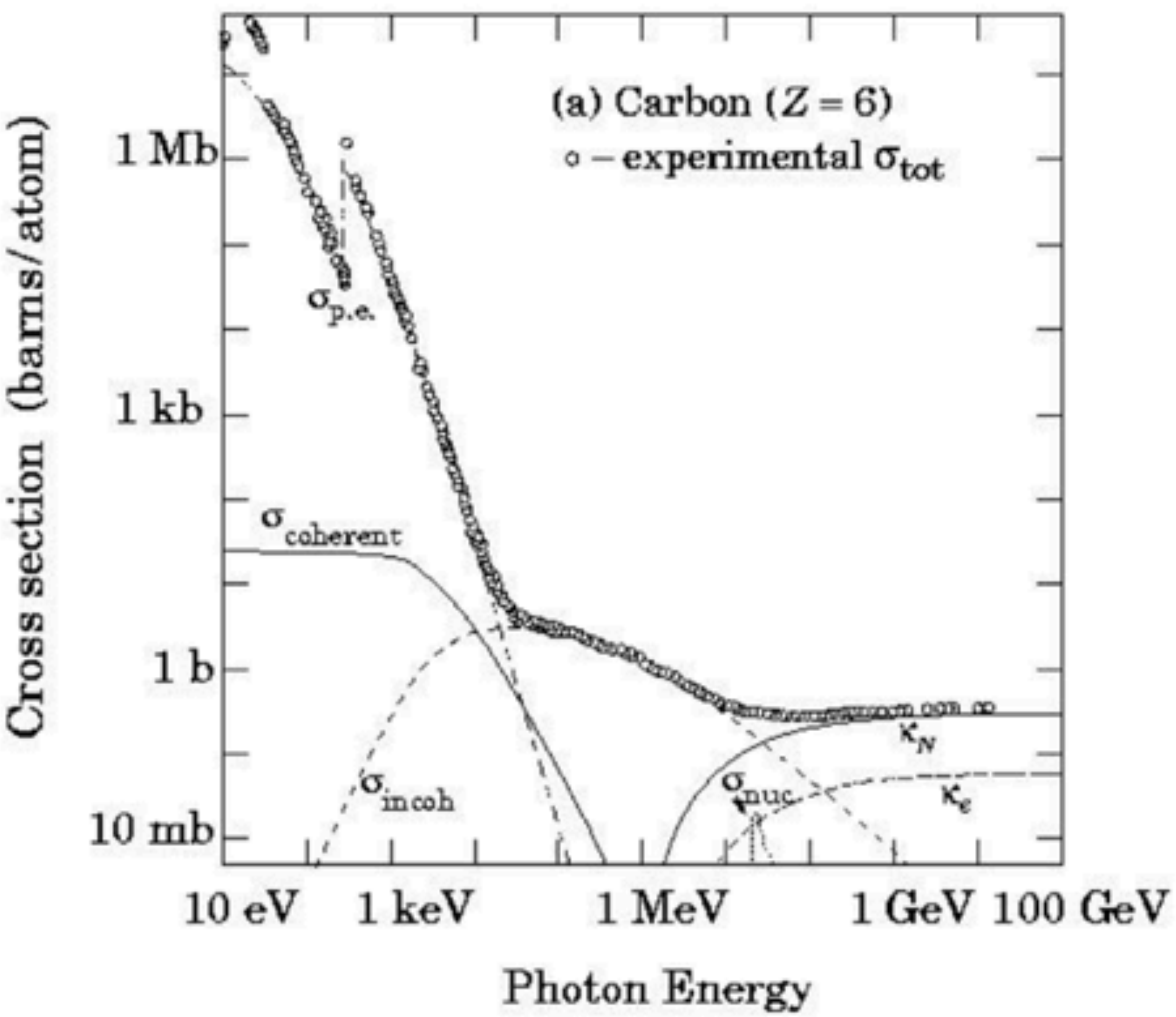
Probability of pair creation in 1 X_0 is $e^{-7/9}$, mean free path of a photon before creating a e^+e^- pair is $\Lambda_{pair} = 9/7\ X_0$

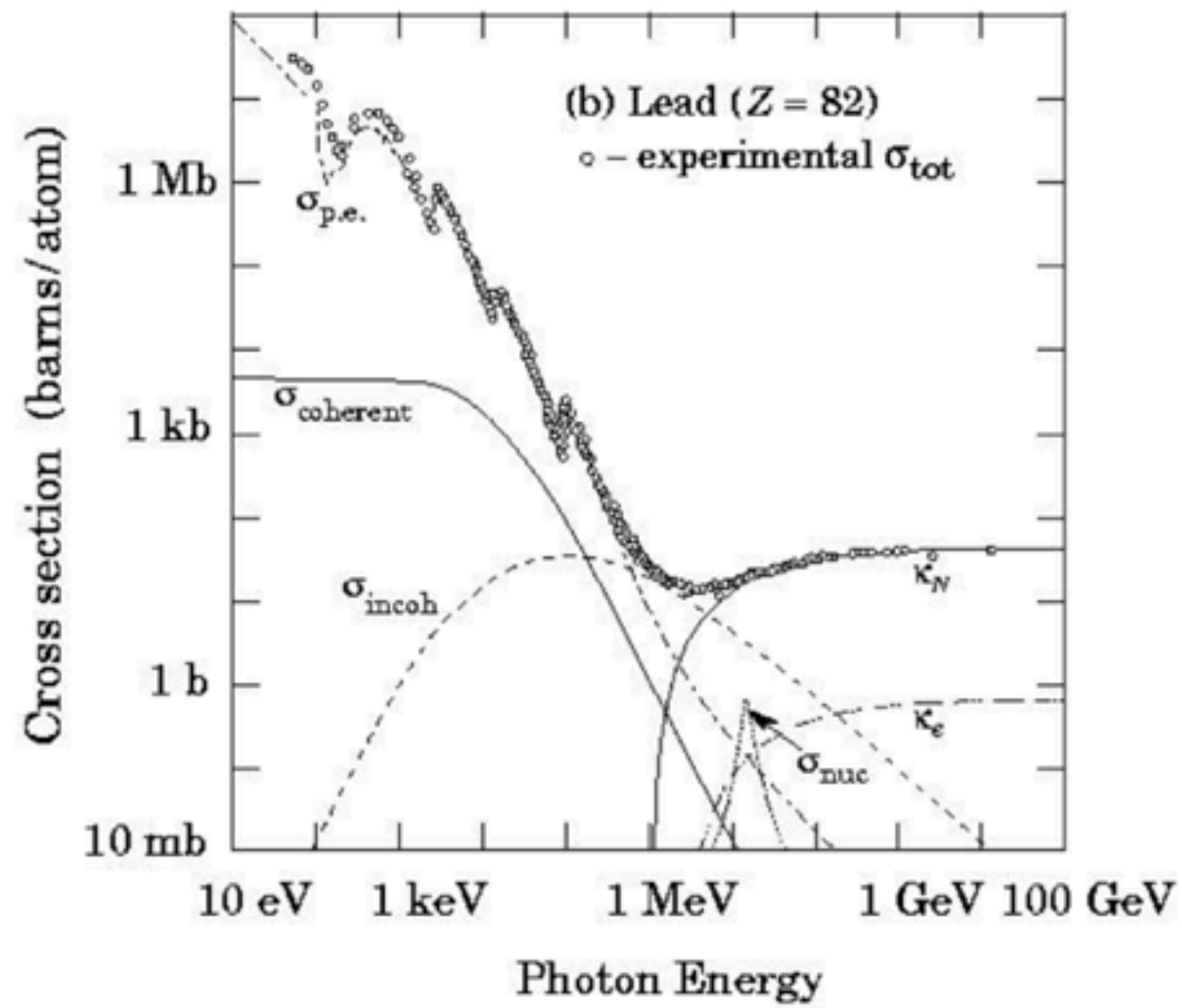
Relative importance



* Low energy and high Z → photoelectric effect
* Medium energy and Z → Compton scattering
* High energy and Z → pair production

Cross section





$\sigma_{\text{p.e.}}$ = Atomic photoelectric effect (electron ejection, photon absorption)

σ_{Rayleigh} = Rayleigh (coherent) scattering–atom neither ionized nor excited

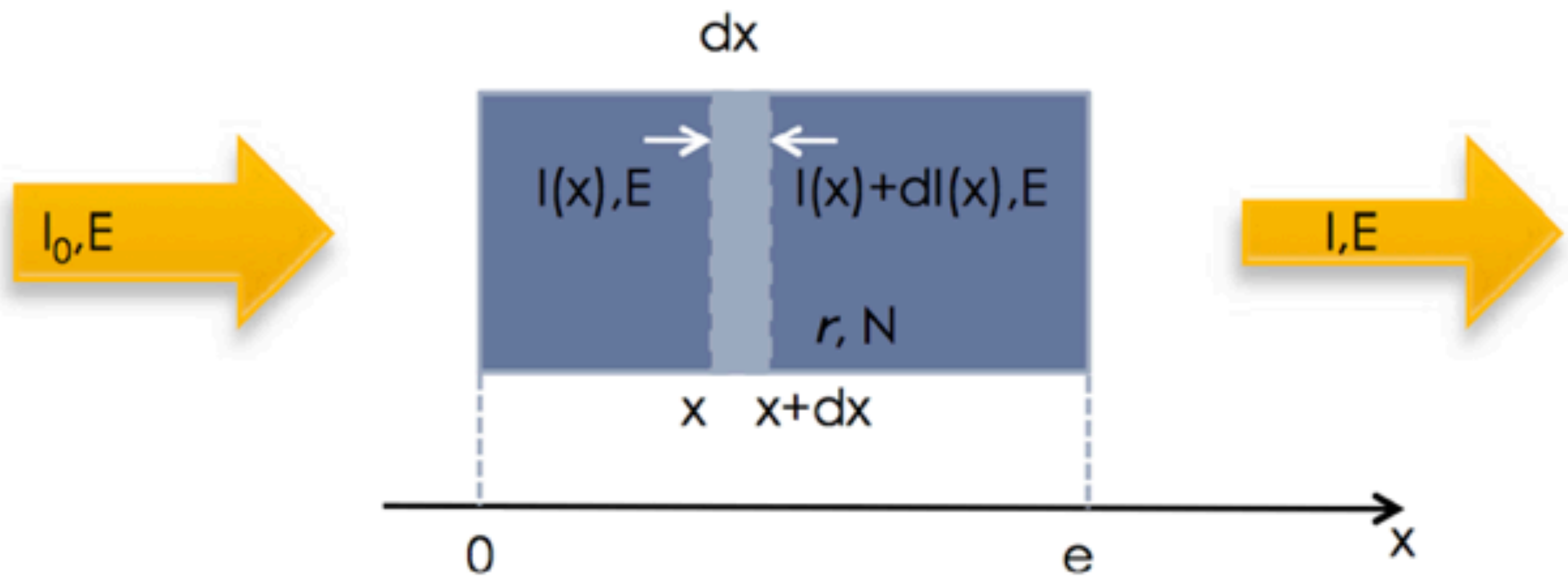
σ_{Compton} = Incoherent scattering (Compton scattering off an electron)

κ_{nuc} = Pair production, nuclear field

κ_e = Pair production, electron field

$\sigma_{\text{g.d.r.}}$ = Photonuclear interactions

Photon attenuation in matter



$dI(x) = -I(x)\,\sigma_T(E)\,N\,dx$ with $N = \frac{N_a \rho}{A}$

where:

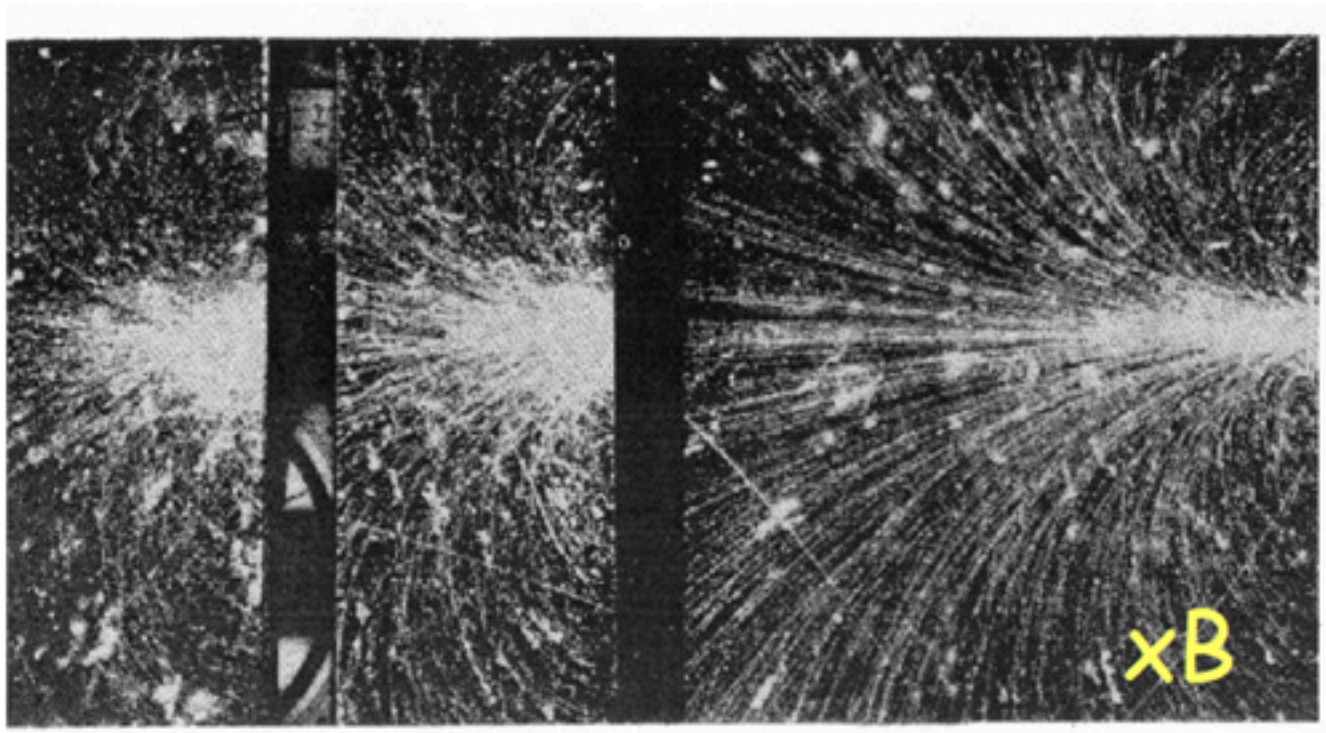
- σ_T is the total cross section and $\sigma_T = \sigma_{photo} + \sigma_{compton} + \sigma_{pair}$
- A is the molecular weight of the media
- ρ is the density of the material [g/cm^3]
- N_a is the Avogadro's number

then

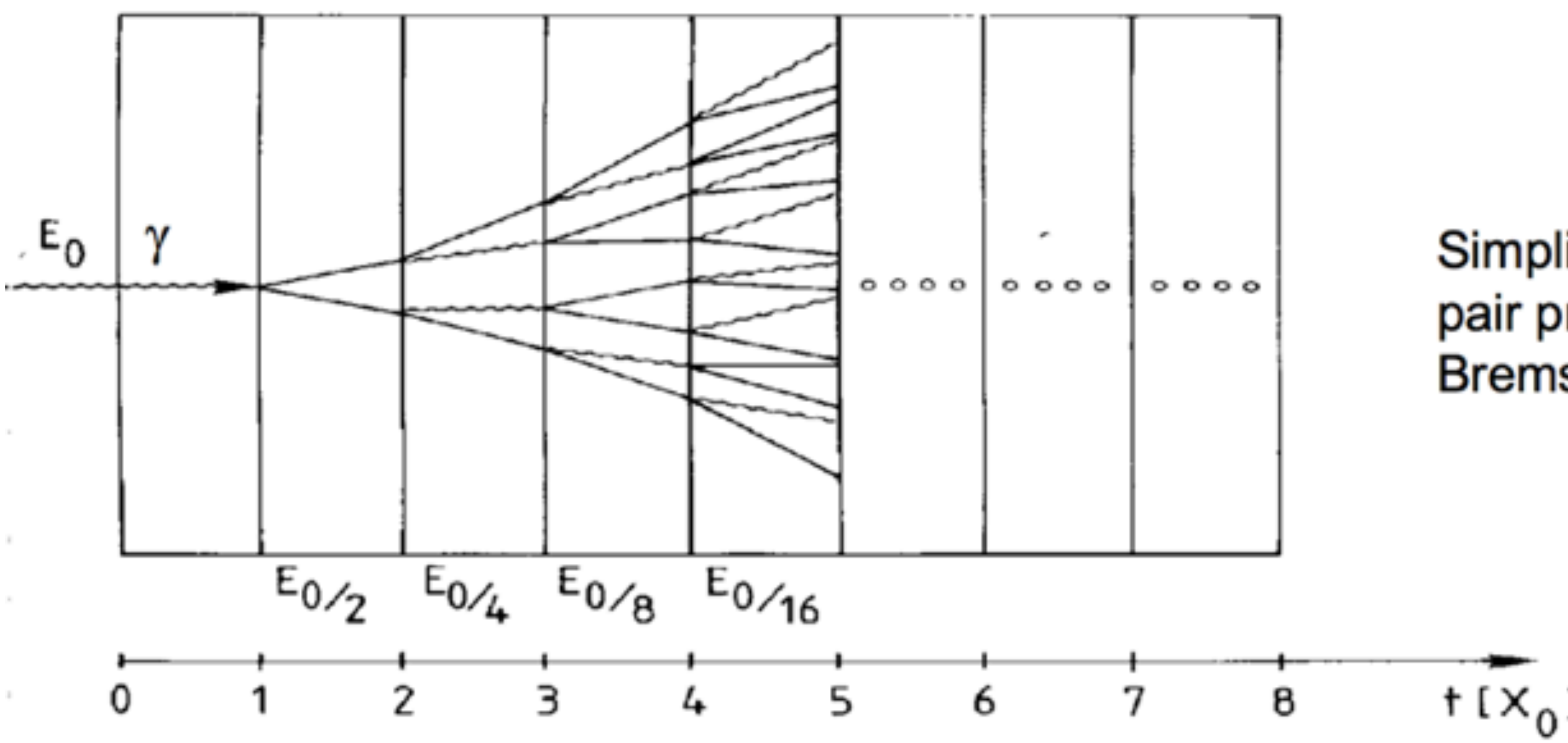
$I(x) = I_0\,e^{-N\sigma_T x} = I_0\,e^{-\mu x}$ and μ is the total absorption coefficient [cm^{-1}]

$(\mu_m)_c = \left(\frac{\mu}{\rho}\right)_c = \sum_i w_i \left(\frac{\mu}{\rho}\right)_i$ where w_i is the weight fraction and $\left(\frac{\mu}{\rho}\right)_i$ the mass attenuation coefficient [cm^2/g] of each element in the compound.

Electromagnetic shower



Electron shower in lead. 7500 gauss in cloud chamber. CALTECH

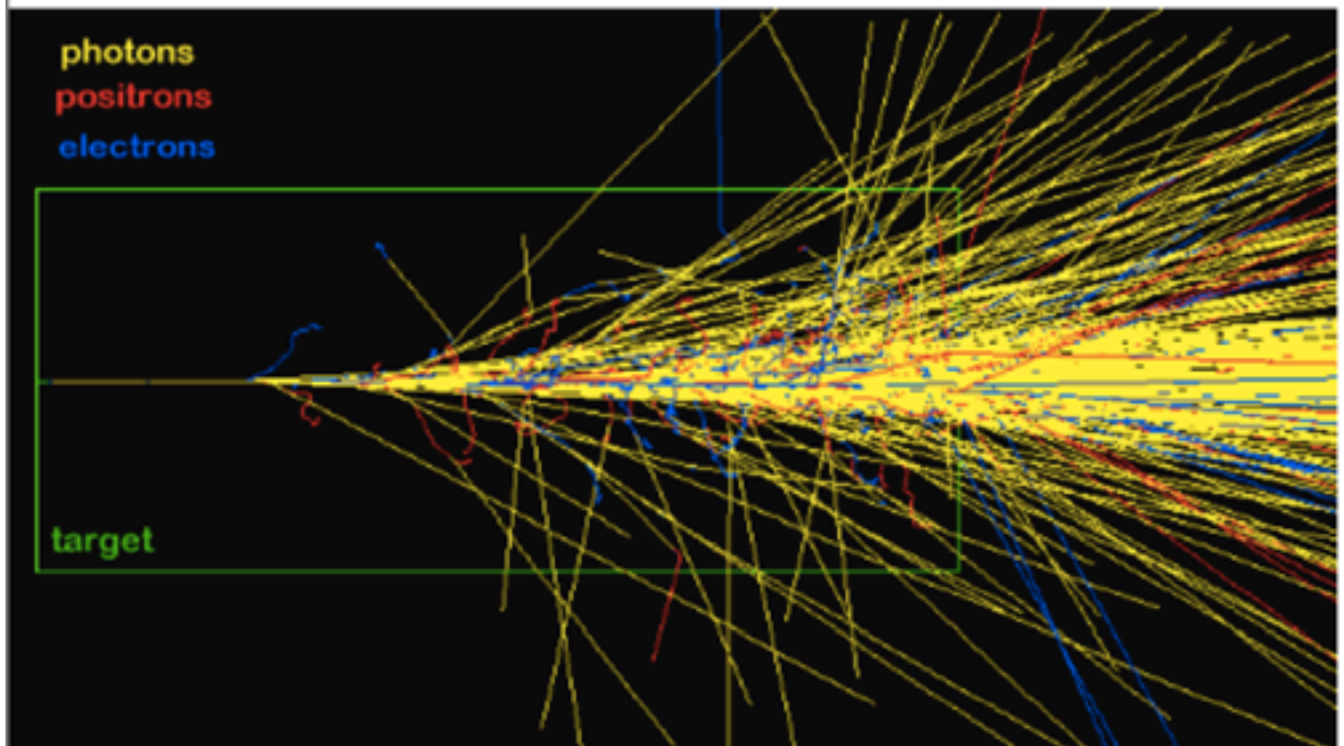


Simplified model, considering <u>only</u> pair production and Bremsstrahlung (+ hyp. : $X_0 = \lambda_{pair}$)

$N(t) = 2^t \qquad E(t)/\,particle = E_0 \cdot 2^{-t}$

Moliere radius

Massive shower in a tungsten cylinder (outlined in green) produced by a single 10 GeV incident electron.



Transverse profile increases:

- e^+e^- production angle
- e multiple scattering
- Emission of photons

- Transverse dimensions of electromagnetic showers are measured in terms of the *Moliere radius*:

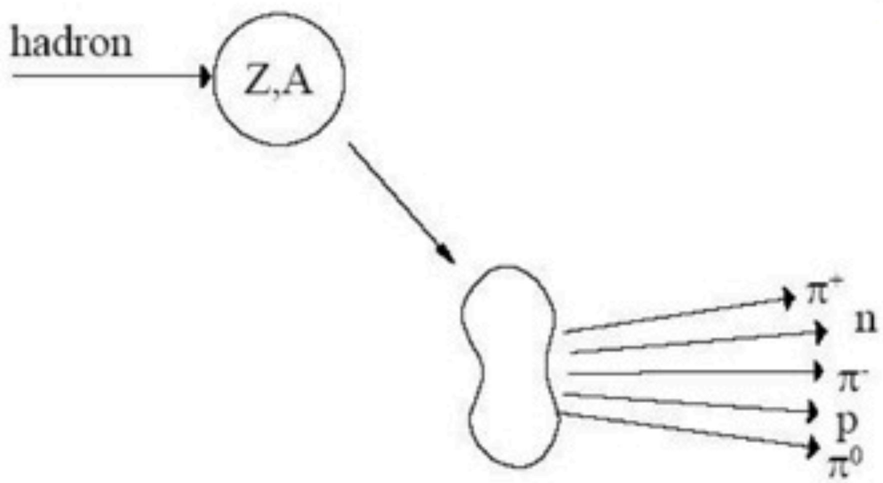
$$R_M = \frac{21\ \text{MeV}}{E_c} X_0$$

with E_c the critical energy

- More than 90% of the shower energy is contained in $\pm\, 2R_M$
- R_M (Pb)=(21/9,51) x 0,56 = 1,24 cm

Interaction of hadrons

Interaction of energetic hadrons (charged/neutral) through matter involves nuclear interaction :
excitation and nucleus break up => production of secondary particles + fragment

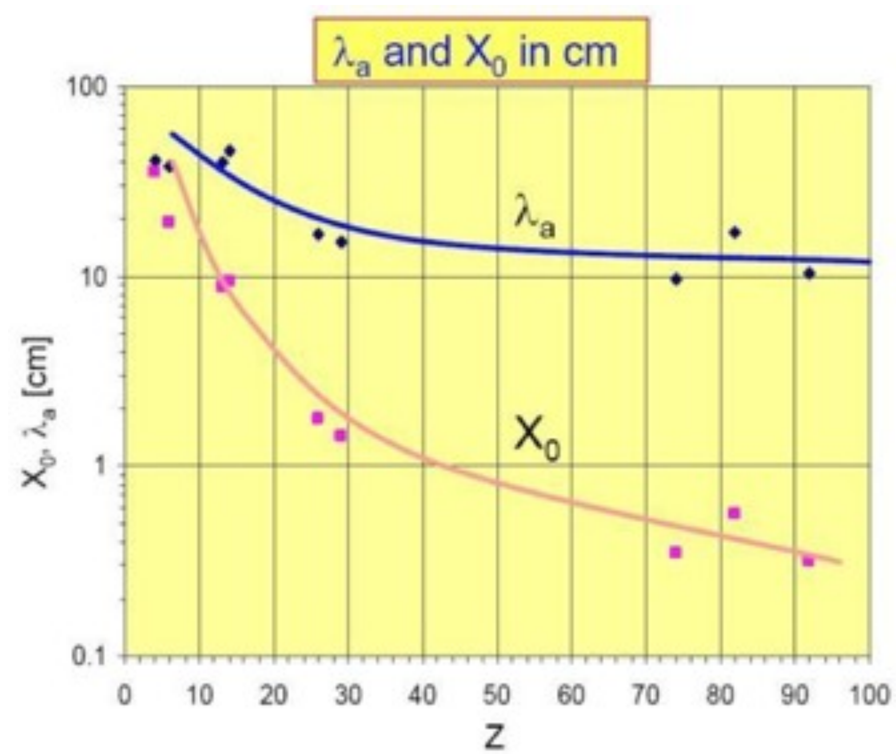


Number of particle produced ~ln (E) with average transverse p of 0.35 GeV/c

For E > 1 GeV, $\sigma \sim \sigma_0 A^{0.7}$, with σ_0= 35 mb and independent of particle type π,p,K,...
Convenient to introduce the hadronic interaction (absorption) length :

$$\lambda_{I(a)} = \frac{A}{N_A \sigma_{total(inel)}} \propto A^{1/3}$$

the mean distance before nuclear interaction



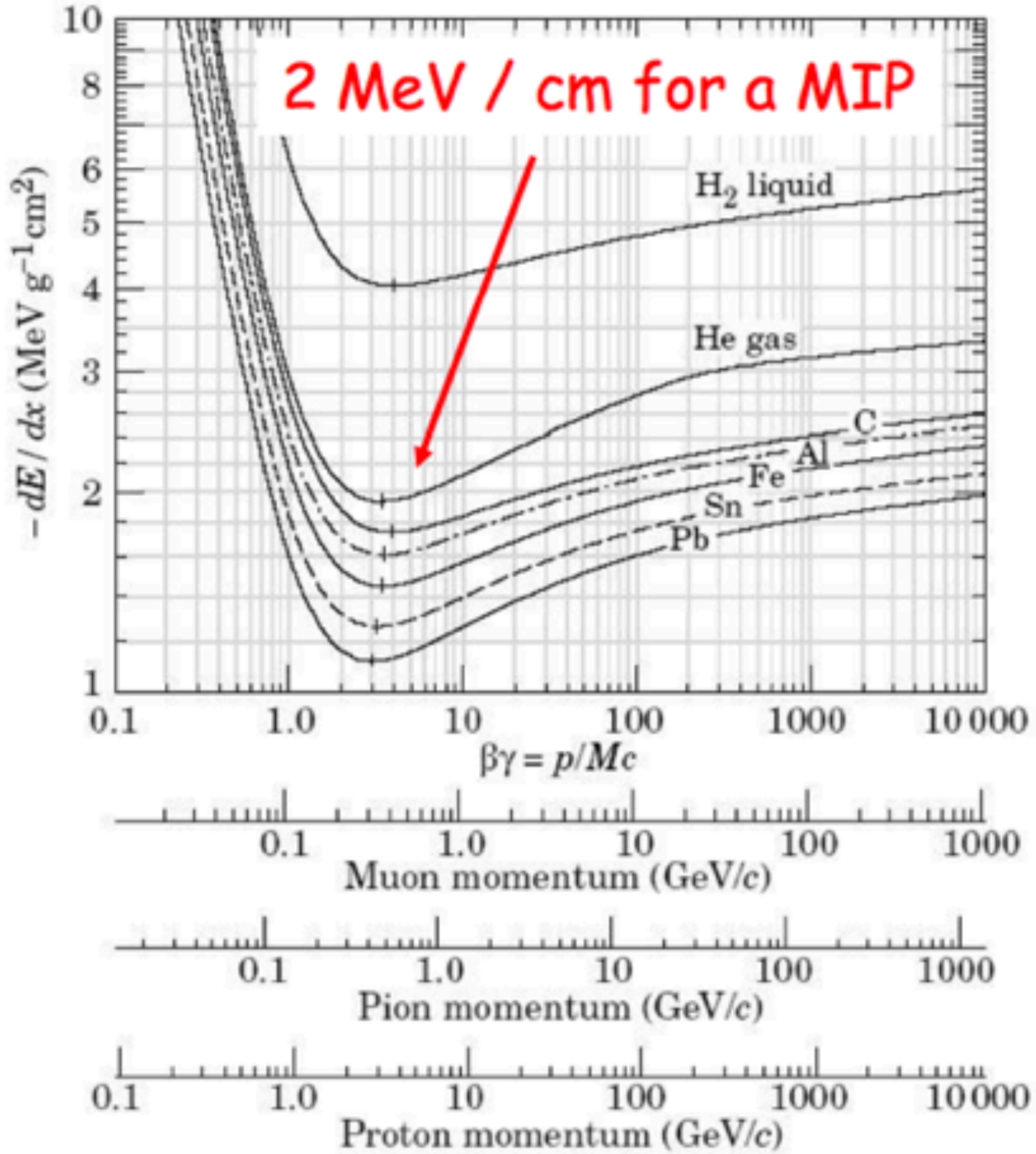
6. ATOMIC AND NUCLEAR PROPERTIES OF MATERIALS

Table 6.1 Abridged from **`pdg.lbl.gov/AtomicNuclearProperties`** by D. E. Groom (2007). See web pages for more detail about entries in this table including chemical formulae, and for several hundred other entries. Quantities in parentheses are for NTP (20° C and 1 atm), and square brackets indicate quantities evaluated at STP. Boiling points are at 1 atm. Refractive indices n are evaluated at the sodium D line blend (589.2 nm); values $\gg$1 in brackets are for $(n-1)\times 10^6$ (gases).

Material	Z	A	$\langle Z/A \rangle$	Nucl.coll. length λ_T {g cm^{-2}}	Nucl.inter. length λ_I {g cm^{-2}}	Rad.len. X_0 {g cm^{-2}}	$dE/dx\vert_{\rm min}$ { MeV g^{-1}cm^2}	Density {g cm^{-3}} ({gℓ^{-1}})	Melting point (K)	Boiling point (K)	Refract. index (@ Na D)
H_2	1	1.00794(7)	0.99212	42.8	52.0	63.04	(4.103)	0.071(0.084)	13.81	20.28	1.11[132.]
D_2	1	2.01410177803(8)	0.49650	51.3	71.8	125.97	(2.053)	0.169(0.168)	18.7	23.65	1.11[138.]
He	2	4.002602(2)	0.49967	51.8	71.0	94.32	(1.937)	0.125(0.166)		4.220	1.02[35.0]
Li	3	6.941(2)	0.43221	52.2	71.3	82.78	1.639	0.534	453.6	1615.	
Be	4	9.012182(3)	0.44384	55.3	77.8	65.19	1.595	1.848	1560.	2744.	
C diamond	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.725	3.520			2.42
C graphite	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.742	2.210			
N_2	7	14.0067(2)	0.49976	61.1	89.7	37.99	(1.825)	0.807(1.165)	63.15	77.29	1.20[298.]
O_2	8	15.9994(3)	0.50002	61.3	90.2	34.24	(1.801)	1.141(1.332)	54.36	90.20	1.22[271.]
F_2	9	18.9984032(5)	0.47372	65.0	97.4	32.93	(1.676)	1.507(1.580)	53.53	85.03	[195.]
Ne	10	20.1797(6)	0.49555	65.7	99.0	28.93	(1.724)	1.204(0.839)	24.56	27.07	1.09[67.1]
Al	13	26.9815386(8)	0.48181	69.7	107.2	24.01	1.615	2.699	933.5	2792.	
Si	14	28.0855(3)	0.49848	70.2	108.4	21.82	1.664	2.329	1687.	3538.	3.95
Cl_2	17	35.453(2)	0.47951	73.8	115.7	19.28	(1.630)	1.574(2.980)	171.6	239.1	[773.]
Ar	18	39.948(1)	0.45059	75.7	119.7	19.55	(1.519)	1.396(1.662)	83.81	87.26	1.23[281.]
Ti	22	47.867(1)	0.45961	78.8	126.2	16.16	1.477	4.540	1941.	3560.	
Fe	26	55.845(2)	0.46557	81.7	132.1	13.84	1.451	7.874	1811.	3134.	
Cu	29	63.546(3)	0.45636	84.2	137.3	12.86	1.403	8.960	1358.	2835.	
Ge	32	72.64(1)	0.44053	86.9	143.0	12.25	1.370	5.323	1211.	3106.	
Sn	50	118.710(7)	0.42119	98.2	166.7	8.82	1.263	7.310	505.1	2875.	
Xe	54	131.293(6)	0.41129	100.8	172.1	8.48	(1.255)	2.953(5.483)	161.4	165.1	1.39[701.]
W	74	183.84(1)	0.40252	110.4	191.9	6.76	1.145	19.300	3695.	5828.	
Pt	78	195.084(9)	0.39983	112.2	195.7	6.54	1.128	21.450	2042.	4098.	
Au	79	196.966569(4)	0.40108	112.5	196.3	6.46	1.134	19.320	1337.	3129.	
Pb	82	207.2(1)	0.39575	114.1	199.6	6.37	1.122	11.350	600.6	2022.	
U	92	[238.02891(3)]	0.38651	118.6	209.0	6.00	1.081	18.950	1408.	4404.	
Air (dry, 1 atm)			0.49919	61.3	90.1	36.62	(1.815)	(1.205)		78.80	
Shielding concrete			0.50274	65.1	97.5	26.57	1.711	2.300			

Summary

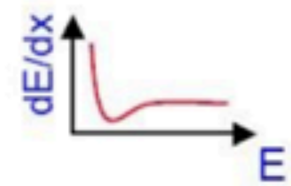
Bethe-Bloch for heavy charged particles



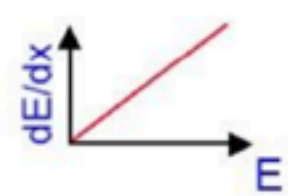
Radiation length X_0

e^+ / e^-

- Ionisation



- Bremsstrahlung

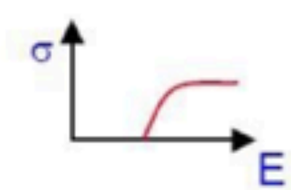


γ

- Photoelectric effect

- Compton effect

- Pair production



Interaction of hadrons : many different particles produced,

interaction length λ_I