Radiative processes in High-Energy astrophysics

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OUTLINE

Special relativity

Doppler effects (beaming)

Synchrotron

Compton scattering

http://arxiv.org/archive/astro-ph

13. arXiv:1202.5949 [pdf, ps, other]

Radiative Processes in High Energy Astrophysics

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Comments: 157 pages, 64 figures. Lecture notes for a university course

Subjects: **High Energy Astrophysical Phenomena (astro-ph.HE)**

Luminosity L (bolometric):

Flux F (bolometric): energy passing a surface of $1 cm^2$ per second [$erg/cm^2/s$]

- monochromatic $F(\nu)$ [$erg/cm^2/s/Hz$]

- in a given energy range $F_{[\nu 1-\nu 2]}$ [$erg/cm^2/s$]

$$F = \frac{L}{4\pi R^2}; \quad F(\nu) = \frac{L(\nu)}{4\pi R^2};$$

Fluence S: energy passing a surface of $1 cm^2$ integrated over the duration of the emission [erg/cm^2]

T = duration of the emission

$$S = \int_0^T F(t)dt$$

If F(t)=constant

$$S = F \cdot T$$

Exercises for tomorrow:

1. estimate the flux of the Sun on the Earth (pg. 7-8)

2. estimate the flux of a GRB with $L=10^{52}$erg/s at $z=2$ (pg. 8)

3. estimate the fluence of the GRB in exercise 2 assuming that the flux is constant and the emission lasts 20 seconds. How much time does it take to collect the same fluence from the Sun?

Special relativity

Consider a ruler and a clock both moving with velocity v.
We can define two different reference frames:

1. K that sees the ruler and the clock moving at velocity v
2. K' that sees the ruler and the clock at rest

For simplicity, we consider a motion along the x-axis

$$\beta = \frac{v}{c} \qquad \Gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Special relativity: length contraction $\Delta x = \frac{\Delta x'}{\Gamma} \rightarrow \text{contraction}$

time dilation $\Delta t = \Gamma \Delta t' \rightarrow \text{dilation}$

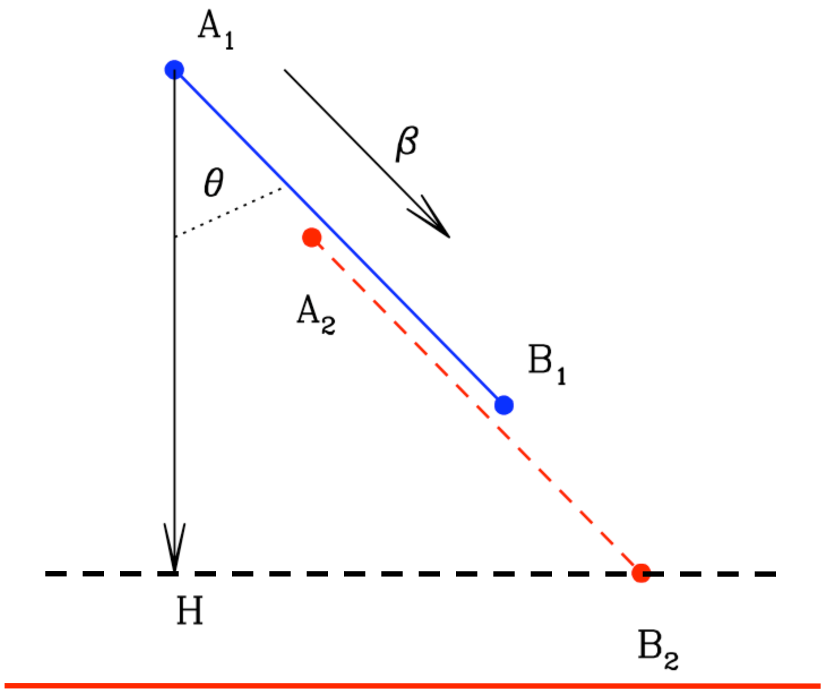
Exercises for tomorrow:

4. estimate β and Γ (the Lorentz factor) of an object moving at $v=10^{10}$ cm/s. Is this object moving at a relativistic velocity? (relativistic velocity=Γ is appreciably different than 1)

5. estimate the velocity v and β of a parcel of matter moving with a Lorentz factor $\Gamma=100$ (typical Lorentz factor of the fluid in GRBs)

Let's now take a picture of the ruler!
Picture (or detector): collects photons arriving at the same time, but not necessarily emitted at the same time!

Consider an extended object (a bar) moving with velocity βc and reflecting (or emitting) photons.
l'=proper length $l=l'/\Gamma$



The photon emitted in A1 at $t=t_i$ after a time Δt_e reaches H. In the meantime, the bar moves from its initial position A_1B_1 to the final one A_2B_2. The photon emitted in B_2 reaches the detector at the same time of the photon emitted at earlier times in A_1.

Let's now take a picture of the ruler!
Picture (or detector): collects photons arriving at the same time, but not necessarily emitted at the same time!

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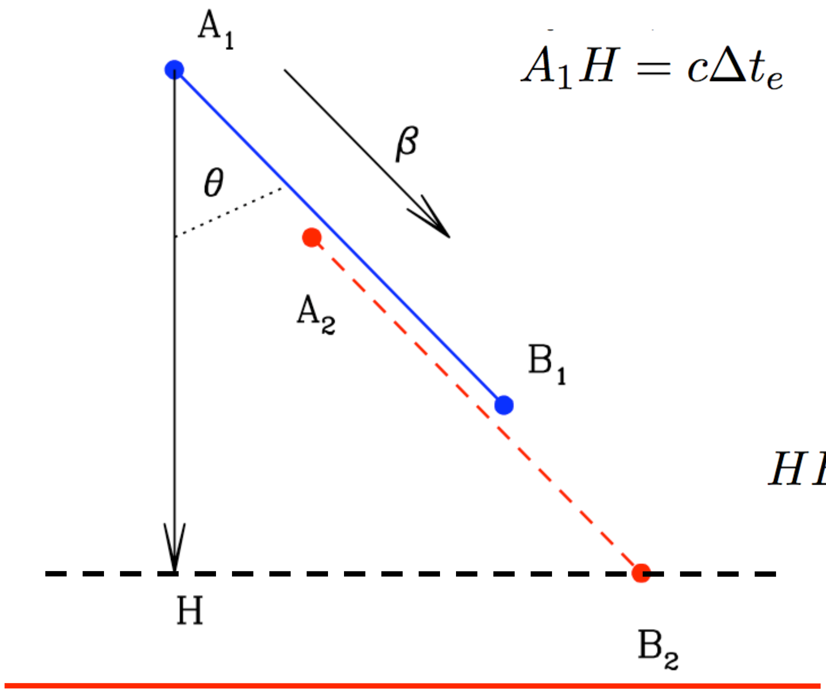


$$A_1H = c\Delta t_e \qquad A_1B_1 = \frac{\ell'}{\Gamma} \qquad B_1B_2 = \beta c \Delta t_e$$

$$A_1B_2 = \frac{A_1H}{\cos\theta} = \frac{\ell'}{\Gamma(1-\beta\cos\theta)}$$

$$HB_2 = A_1B_2\sin\theta = \ell'\frac{\sin\theta}{\Gamma(1-\beta\cos\theta)} = \ell'\delta\sin\theta$$

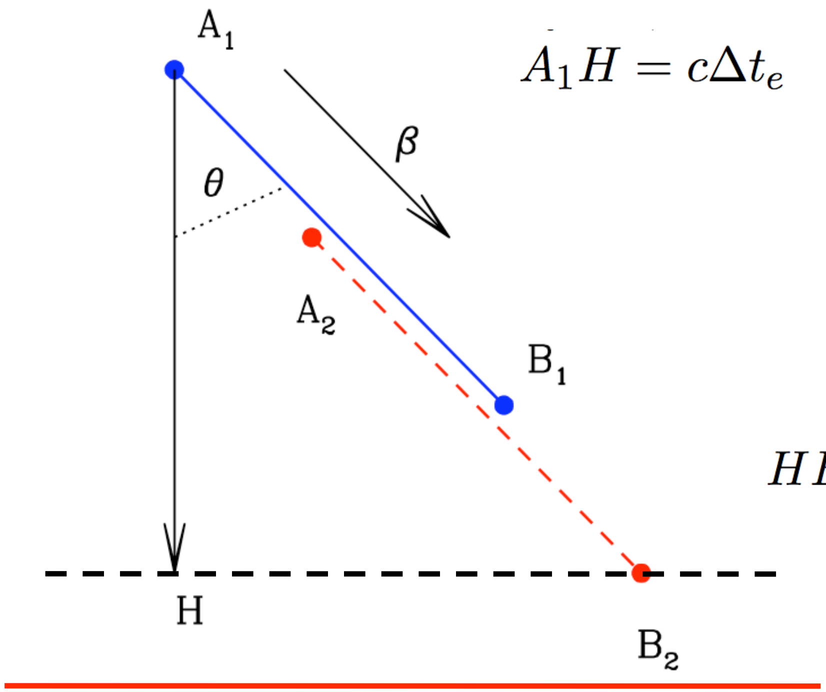
Definition $\delta = 1/[\Gamma(1-\beta\cos\theta)]$

The observed length depends on the viewing angle:

- reaches the maximum (equal to l') for $\cos\theta=\beta$
- is equal to l'/Γ for $\theta=90°$
- is zero for $\theta=0°$

To keep:

- viewing angle (between direction of photons reaching the observer and the velocity of the source of photons) is important
- distinguish between emission time and arrival time



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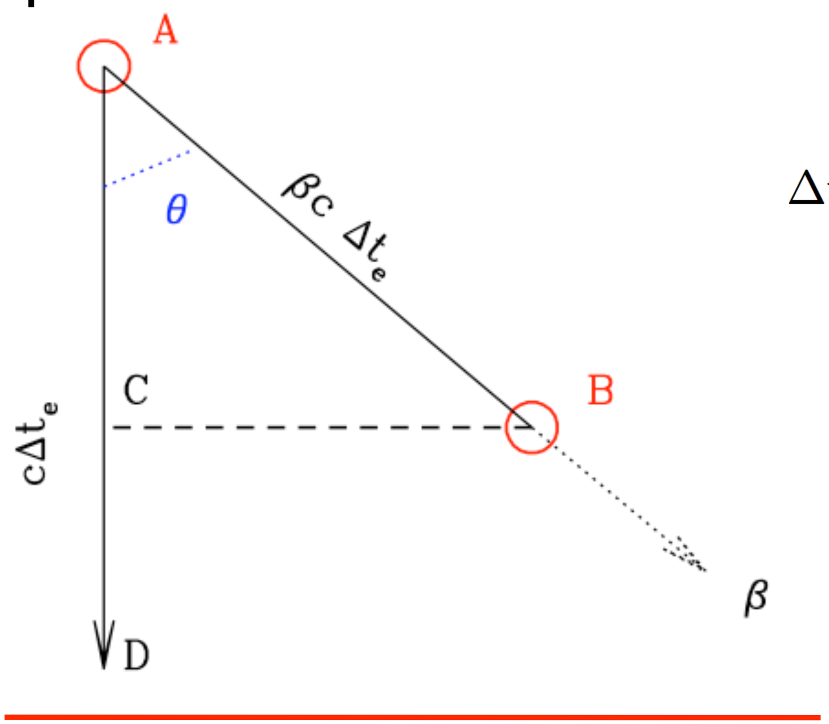
Definition $\delta = 1/[\Gamma(1-\beta\cos\theta)]$

Exercises for tomorrow:

6. Figure 3.1 in Ghisellini 2012: demonstrate that the observed length HB_2 (see eq. 3.8) reaches a maximum for $\cos\theta=\beta$ and that this maximum length is equal to l'.

Consider the following situation:
relativistic electron emitting radiation

Electron starts to emit when it is in A and stops when it reaches B. The difference between emission times is Δt_e. The first photon (emitted at A) after Δt_e reaches D. The electron instead, after Δt_e reaches B and emits the last photon. What is the difference in the arrival times Δt_a?



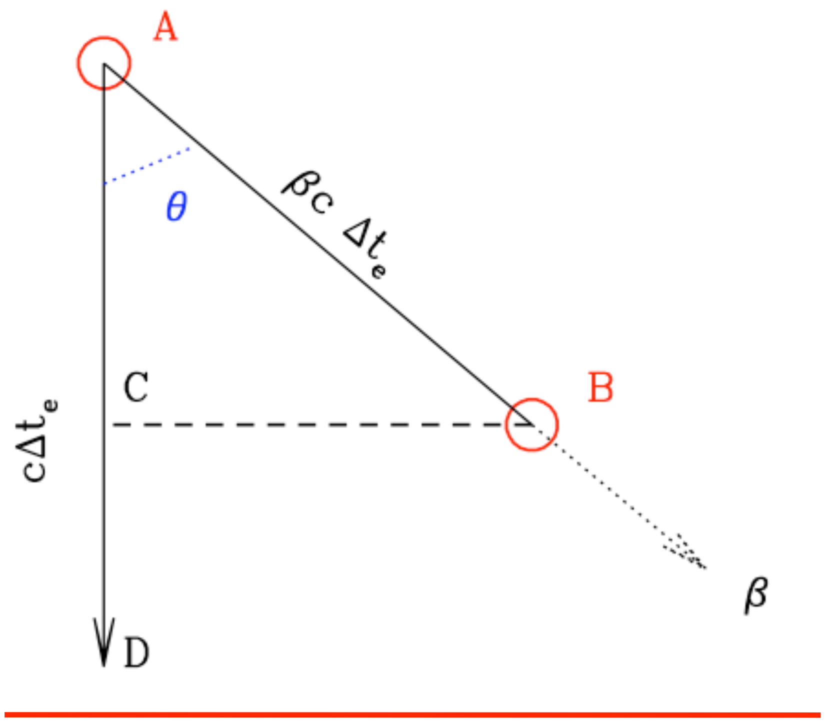
$$\begin{aligned} \Delta t_{\mathrm{a}} &= \frac{CD}{c} = \frac{AD - AC}{c} = \Delta t_{\mathrm{e}} - \beta \Delta t_{\mathrm{e}} \cos\theta \\ &= \Delta t_{\mathrm{e}}(1 - \beta\cos\theta) \\ &= \Delta t'_{\mathrm{e}} \Gamma (1 - \beta\cos\theta) \\ &= \frac{\Delta t'_{\mathrm{e}}}{\delta} \end{aligned}$$

$$\Delta t_{\rm a} = \Delta t'_{\rm e}\Gamma(1-\beta\cos\theta)$$

For θ=0° (electron is moving toward us)

$$\Delta t_{\rm a} = \Delta t'_{\rm e}\Gamma(1-\beta) = \Delta t'_e\Gamma\frac{(1-\beta^2)}{1+\beta} = \Delta t'_e\Gamma\frac{1}{\Gamma^2(1+\beta)} = \frac{\Delta t'_e}{\Gamma(1+\beta)}$$

Time contraction!



For θ=90°

$$\Delta t_{\rm a} = \Delta t'_{\rm e}\Gamma$$

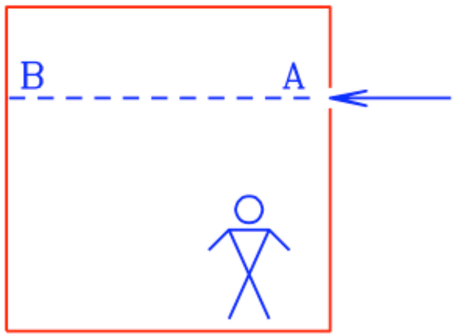
Time dilation = usual special relativity (Lorentz transformations)

Aberration of light

Another very important effect occurring when a source is moving at relativistic velocities is aberration of light.

Trajectory of the photon appears inclined

Angles are different in different frames



$$\sin\theta = \frac{\sin\theta'}{\Gamma(1+\beta\cos\theta')}$$

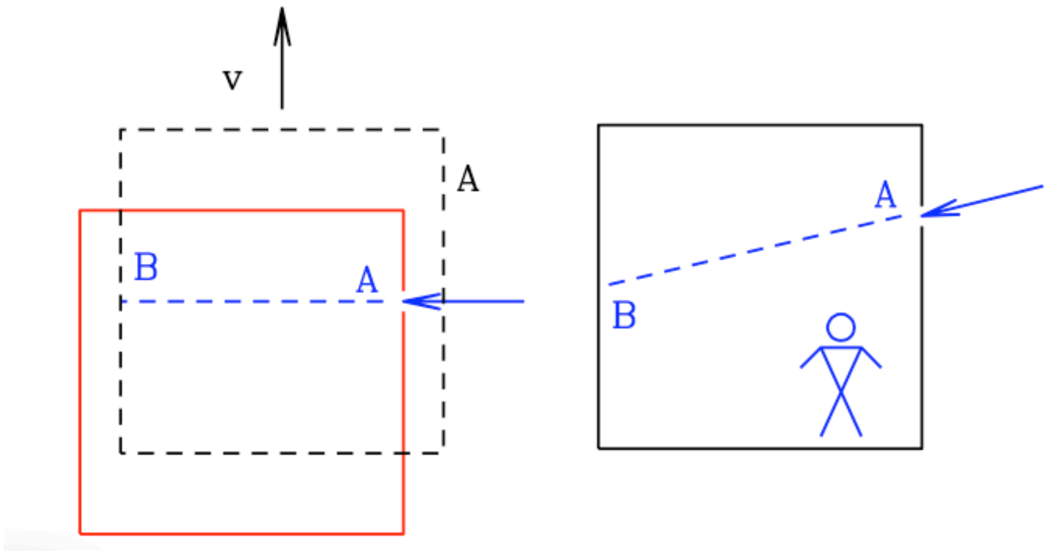
$$\cos\theta = \frac{\cos\theta' + \beta}{1+\beta\cos\theta'};$$

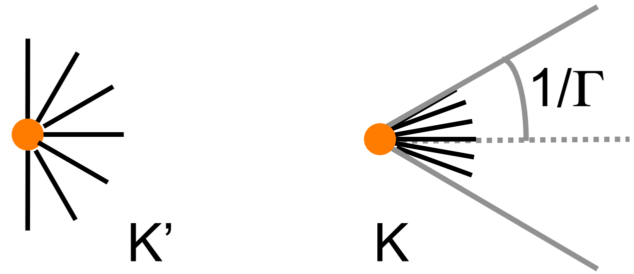
For θ'=90° sinθ=1/Γ

If Γ>>1 then sinθ≈θ

Isotropic source emits half of its photons at θ'<90°

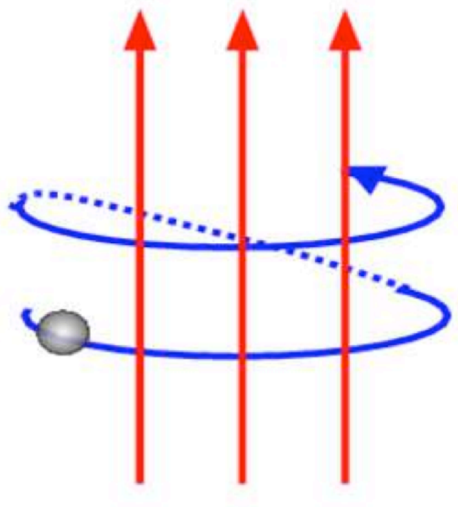
Observer sees half of photons beamed in a cone of semiaperture 1/Γ





Synchrotron emission

Two ingredients: relativistic particles and magnetic field
What is responsible for this kind of radiation is the Lorentz force, making the particle to gyrate around magnetic field lines: change in velocity direction = acceleration = radiation
The velocity modulus does not change, because the Lorentz force does not work.



$$F_{\mathrm{L}} = \frac{d}{dt}(\gamma m \mathbf{v}) = \frac{e}{c}\mathbf{v} \times \mathbf{B}$$

$$r_{\mathrm{L}} = \frac{v_{\perp}^2}{a_{\perp}} = \frac{\gamma m c^2 \beta \sin\theta}{eB}$$

Total power emitted by a single particle with pitch angle θ:

$$P_{\rm S} = \frac{2e^4}{3m^2c^3}B^2\gamma^2\beta^2\sin^2\theta$$

- The magnetic energy density is $U_B \equiv B^2/(8\pi)$
- the quantity $e^2/(m_{\rm e}c^2)$, in the case of electrons, is the classical electron radius r_0
- the square of the electron radius is proportional to the Thomson scattering cross section $\sigma_{\rm T}$, i.e. $\sigma_{\rm T} = 8\pi r_0^2/3 = 6.65 \times 10^{-25}\ {\rm cm}^2$.

Total power emitted by the single electron:

$$P_{\rm S}(\theta) = 2\sigma_{\rm T}cU_B\gamma^2\beta^2\sin^2\theta$$

For isotropic distribution of pitch angles:

$$\langle P_{\rm S}\rangle = \frac{4}{3}\sigma_{\rm T}cU_B\gamma^2\beta^2$$

Synchrotron cooling time

$$t_{\rm syn} = \frac{E}{\langle P_{\rm S} \rangle} = \frac{\gamma m_{\rm e} c^2}{(4/3)\sigma_{\rm T} c U_B \gamma^2 \beta^2} = \frac{3 m_e c^2}{4 \sigma_{\rm T} c \gamma U_B};$$

Exercises for tomorrow:

7. Estimate the synchrotron cooling time of an electron emitting gamma-rays in a GRB (γ=200 and B=10^6Gauss) and compare it with the cooling time of an electron in the vicinity of a supermassive AGN black hole and in the radio lobes of a radio loud quasar (see section 4.2.1)

Synchrotron spectrum and typical frequency

$$\nu_S = \gamma^2 B \frac{e}{2\pi m_e c}$$

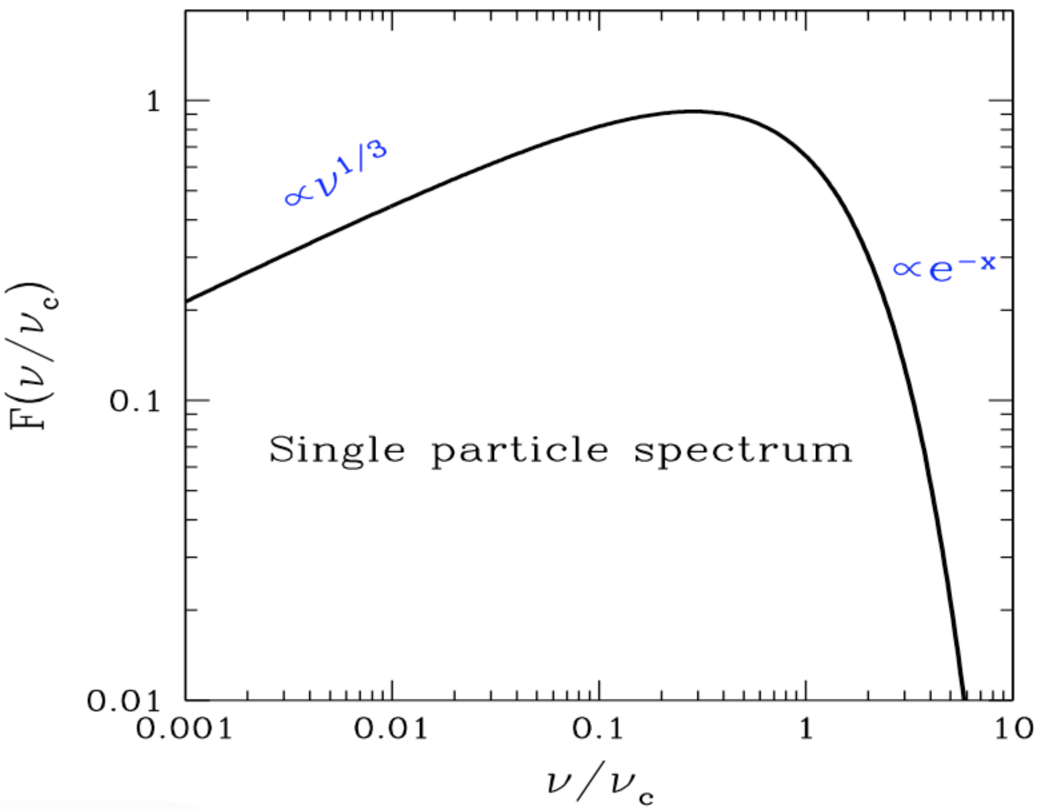
$$\nu_{\rm s} = \gamma^2 \nu_{\rm L}; \qquad \nu_{\rm L} \equiv \frac{eB}{2\pi m_{\rm e} c}$$

Exercises for tomorrow:

8. Estimate the typical synchrotron frequency of the electron in exercise 7
   a) in the frame at rest with the emitting electron
   b) in the frame of the observer that see the electron moving toward him with a Lorentz factor Γ=100.

Synchrotron spectrum and typical frequency

Emission from 1 single electron



Electron energy distribution:

In high-energy astrophysics is often a power-law distribution:

$$N(\gamma) = K\,\gamma^{-p}$$

The resulting spectrum: power-law segments

$$t < t_{\rm cool} \quad \Rightarrow \quad F(\nu) \propto \begin{cases} \nu^2 & \nu < \nu_{\rm a} \\ \nu^{1/3} & \nu_{\rm a} \le \nu < \nu_m \\ \nu^{-\frac{p-1}{2}} & \nu_m \le \nu < \nu_{\rm cool} \\ \nu^{-\frac{p}{2}} & \nu \ge \nu_{\rm cool} \end{cases}$$

$$t > t_{\rm cool} \quad \Rightarrow \quad F(\nu) \propto \begin{cases} \nu^2 & \nu < \nu_{\rm a} \\ \nu^{1/3} & \nu_{\rm a} \le \nu < \nu_{\rm cool} \\ \nu^{-\frac{1}{2}} & \nu_{\rm cool} \le \nu < \nu_m \\ \nu^{-\frac{p}{2}} & \nu \ge \nu_m \end{cases}$$

Compton scattering

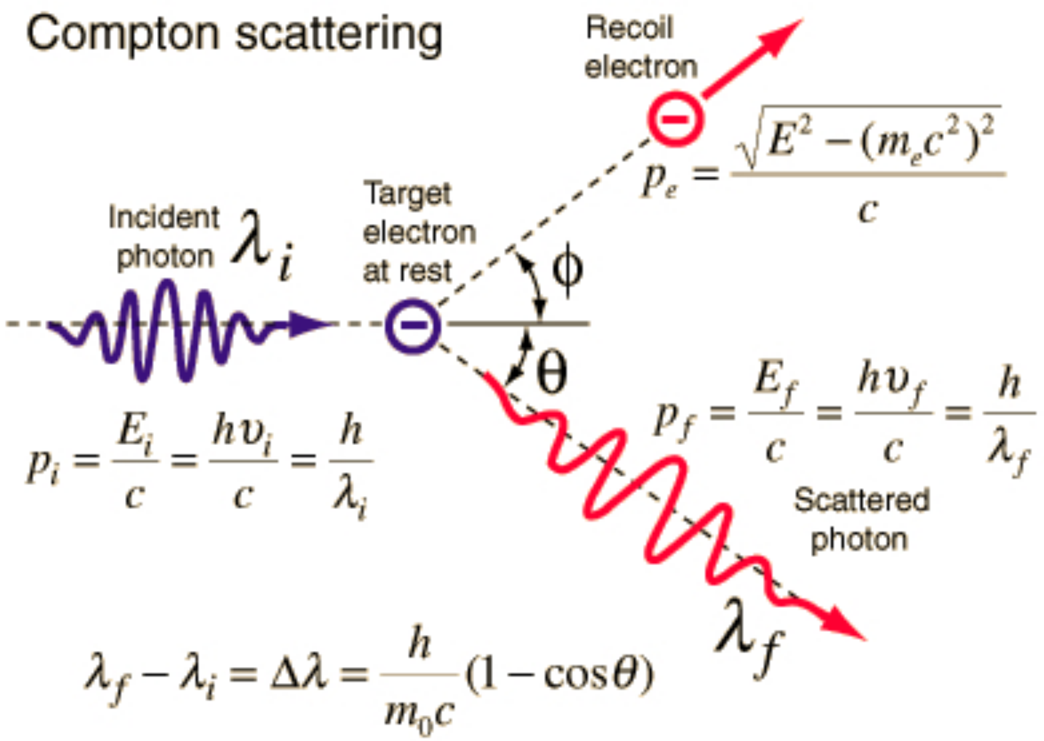
Ingredients: photons and electrons

Direct Compton scattering: when the electron is at rest → transfer of energy from photon to electron

Inverse Compton scattering: electron has a energy (greater than the typical photon energy) → transfer of energy from electron to photon

Direct Compton scattering

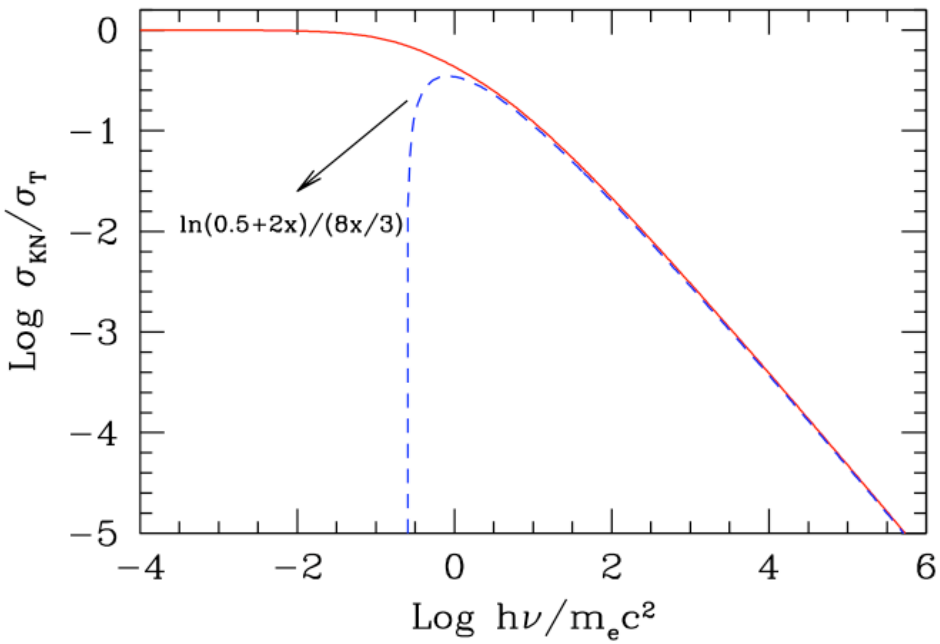
electron at rest and incoming photon



Cross section:

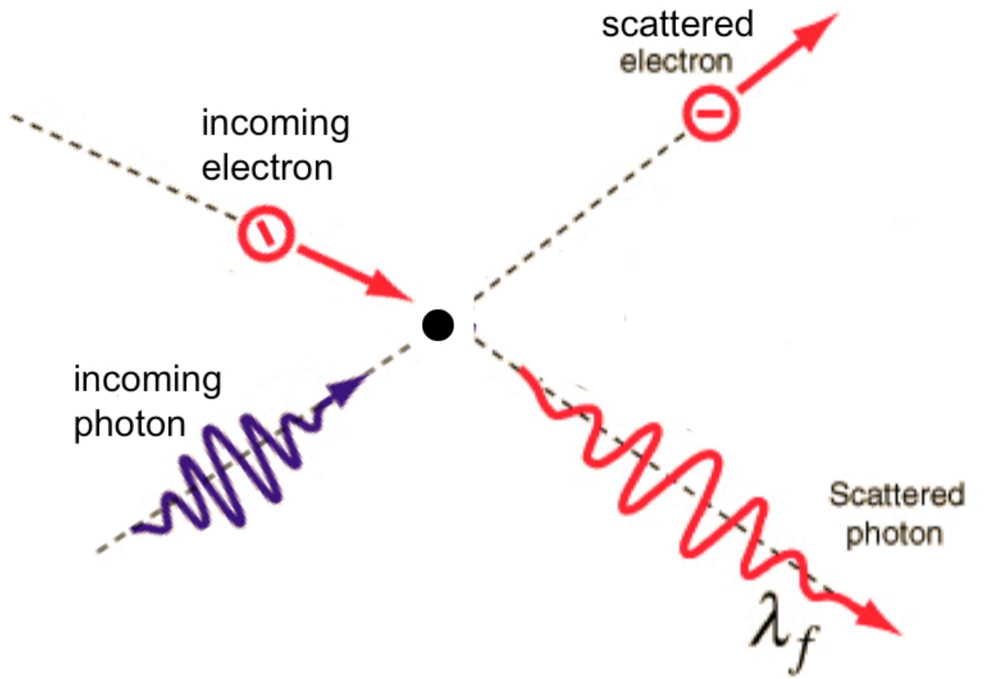
When the energy of the incoming photon (as seen by the electron) is small with respect to $m_e c^2$ the process is called Thomson scattering

When the energy of the incoming photon (as seen by the electron) is comparable or larger then $m_e c^2$ the process in in the Klein-Nishina regime.



Inverse Compton scattering

electron at rest and incoming photon



Final photon energy

$$E_f = \frac{4}{3}\gamma^2 E_i$$

Compton power:

$$P_c(\gamma) = \frac{4}{3}\sigma_T c \gamma^2 \beta^2 U_r$$

Compton cooling time

$$t_{IC} = \frac{3 m_e c^2}{4 \sigma_T c \gamma U_r};$$

Compton:

Final photon energy

$$\nu_f = \frac{4}{3}\gamma^2\nu_i$$

Compton power:

$$P_\mathrm{c}(\gamma) = \frac{4}{3}\sigma_\mathrm{T} c\gamma^2\beta^2 U_\mathrm{r}$$

Compton cooling time

$$t_\mathrm{IC} = \frac{3m_e c^2}{4\sigma_\mathrm{T} c\gamma U_\mathrm{r}};$$

Synchrotron:

$$\nu_\mathrm{s} = \gamma^2\nu_\mathrm{L}$$

$$\langle P_\mathrm{S}\rangle = \frac{4}{3}\sigma_\mathrm{T} c U_B \gamma^2\beta^2$$

$$t_\mathrm{syn} = \frac{3m_e c^2}{4\sigma_\mathrm{T} c\gamma U_B};$$

Synchrotron self Compton:

Population of relativistic electrons in a magnetized region. They produce synchrotron radiation and fill the region with photons. These photons are then up-scattered by the same population of electrons